When to Unbundle Policy Authority

Appendix for online publication

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1 Bundling (Proof of Proposition 1)

Proposition 1. *On the equilibrium path of play under bundling:*

1. *The Agent chooses to exert effort on both tasks if, and only if, either*

   (a) *the complexity of each task is sufficiently low,* $e_i \geq \frac{2k}{e_j B}$*, and the Principal's estimation of the Agent's competence decreases unless the outcome is success on both tasks,* $e_i^H(1-e_j^H) \leq e_i^L(1-e_j^L)$ *for all* $i = 1, 2$*; or*

   (b) *the complexity of each task is moderate,* $1 - \frac{k}{e_j B} \geq e_i \geq \frac{k}{(1-e_j)B}$*, and the Principal's estimation of the Agent's competence increases when the outcome is success on at least one task,* $e_i^H(1-e_j^H) \geq e_i^L(1-e_j^L)$ *for all* $i = 1, 2$.

   In case (a), the Principal adopts the strict retention rule, in case (b), the moderate retention rule.

2. *The Agent chooses to exert effort on a single task* i *when the complexity of that task is sufficiently low,* $e_i \geq k/B$*, and the Principal adopts the* i^{th}*-task retention rule.*[1]

3. *The Agent chooses to exert no effort on either task when the complexity of each task is sufficiently high,* $e_i < k/B$ *for all* $i = 1, 2$*, independent of the Principal's retention rule.*

To prove Proposition 1, we first derive, in Lemma A. 1 the best response of the Principal to effort allocations chosen by the Agent. We then derive, in Lemma A. 2, the Agent's best response effort allocation to retention rules used by the Principal.

[1]It is important to note that, if $e_i < e_j$ the equilibrium with i^{th}-task retention rule is fragile. Indeed, if the Agent deviates to $(a_i = 0, a_j = 1)$ the Principal should be updating favorably on the Agent's type upon observing $o_j = s$ making the deviation profitable. However, the robustness of the i^{th}-task equilibrium is easily assured if we assume that $a_j = 0$ generates $o_j = s$ with arbitrary small probability of success $\epsilon > 0$. Moreover, the question of the equilibrium status of the i^{th}-task action profile when $e_i < e_j$ is irrelevant for our purposes as none of the substantive results discussed in the paper depend on it.

Lemma A. 1 (Best-response of the Principal)**.** *1. If the Principal expects that the Agent chooses* $(a_1 = 1, a_2 = 1)$ *or if the Principal observes that the Agent chooses* $(a_1 = 1, a_2 = 1)$*, then the Principal retains the Agent upon observing* $(o_1 = s, o_2 = s)$*, retains upon observing* $(o_i = s, o_j = f)$ *if, and only if,* $e_i^H(1-e_j^H) \geq e_i^L(1-e_j^L)$ *for all* $i = 1, 2$*, and does not retain upon observing* $(o_1 = f, o_2 = f)$*.*

2. *If the Principal expects that the Agent chooses* $(a_i = 1, a_j = 0)$ *or if the Principal observes that the Agent chooses* $(a_i = 1, a_j = 0)$*, then the Principal retains the Agent if, and only if,* $o_i = s$*.*

3. *If the Principal expects that the Agent chooses* $(a_1 = 0, a_2 = 0)$ *or if the Principal observes that the Agent chooses* $(a_1 = 0, a_2 = 0)$*, then the Principal is indifferent between retaining and dismissing the Agent upon observing any of the outcome pairs.*

Proof of Lemma A. 1*.* We denote $(\mathring{a}_1, \mathring{a}_2)$ the Principal's expectations about the Agent's actions when the Principal does not observe the actions (a_1, a_2) chosen by the Agent. The Principal's posterior belief about the Agent's competence, upon observing outcomes (o_1, o_2) and given expectations $(\mathring{a}_1, \mathring{a}_2)$, is then denoted $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$. Similarly, the Principal's posterior, upon observing outcomes (o_1, o_2) and effort choices (a_1, a_2), is denoted $Pr(\theta = \theta_H|(o_1, o_2); (a_1, a_2))$. Note that we have $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H|(o_1, o_2); (a_1, a_2))$ whenever $(o_1, o_2); (\mathring{a}_1, \mathring{a}_2) = (o_1, o_2); (a_1, a_2)$. To simplify notation, we thus only look at $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$ in the sequel of this proof. Remember that it is a best response for the Principal to retain the Agent if, and only if, $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) \geq \pi$. We have

$$Pr(\theta = \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = \frac{e_1^H e_2^H \pi}{e_1^H e_2^H \pi + e_1^L e_2^L (1-\pi)},$$

$$Pr(\theta = \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = \frac{(1-e_1^H)(1-e_2^H)\pi}{(1-e_1^H)(1-e_2^H)\pi + (1-e_1^L)(1-e_2^L)(1-\pi)},$$

and

$$Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = \frac{e_i^H(1 - e_j^H)\pi}{e_i^H(1 - e_j^H)\pi + e_i^L(1 - e_j^L)(1 - \pi)}.$$

Because $e_i^H > e_i^L$ for all $i = 1, 2$, we have $Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (a_1 = 1, a_2 = 1)) > \pi$ and $Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (a_1 = 1, a_2 = 1)) < \pi$. Consequently, the Principal's best response is to retain upon observing $(o_1 = s, o_2 = s)$, and to dismiss upon observing $(o_1 = f, o_2 = f)$. In turn, $Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = Pr(\theta = \theta_H | (o_i = s, o_j = f); (a_1 = 1, a_2 = 1)) \geq \pi$ if, and only if, $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$. Consequently, the Principal's best response is to retain upon observing $(o_i = s, o_j = f)$ if, and only if, $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$. Similarly, we have

$$Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = \frac{e_i^H \pi}{e_i^H \pi + e_i^L(1 - \pi)},$$

and

$$Pr(\theta = \theta_H | (o_i = f, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = \frac{(1 - e_i^H)\pi}{(1 - e_i^H)\pi + (1 - e_i^L)(1 - \pi)}.$$

Because $e_i^H > e_i^L$ for all $i = 1, 2$, we have $Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = Pr(\theta = \theta_H | (o_i = s, o_j = f); (a_i = 1, a_j = 0)) > \pi$ and $Pr(\theta = \theta_H | (o_i = f, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = Pr(\theta = \theta_H | (o_i = f, o_j = f); (a_i = 1, a_j = 0)) < \pi$. Consequently the Principal's best response is to retain upon observing $(o_i = s, o_j = f)$, and to dismiss upon observing $(o_i = f, o_j = f)$. Outcomes $(o_i = s, o_j = s)$ and $(o_i = f, o_j = s)$ are off the equilibrium path in this case and any retention decision is a best response.

Finally,

$$Pr(\theta = \theta_H | (o_i = f, o_j = f); (\mathring{a}_i = 0, \mathring{a}_j = 0)) = \pi.$$

Outcomes $(o_i = s, o_j = s)$ and $(o_i = f, o_j = s)$ are off the equilibrium path in this case and any retention decision is a best response. □

Lemma A. 2. *1. If the Principal retains the Agent if, and only if,* $(o_1 = s, o_2 = s)$, *then the Agent's best response is to choose* $(a_1 = 1, a_2 = 1)$ *if* $e_1 e_2 B - 2k \geq 0$ *and to choose* $(a_1 = 0, a_2 = 0)$ *if* $e_1 e_2 B - 2k \leq 0$.

2. If the Principal retains the Agent if, and only if, $o_i = s$ *for at least some* $i = 1, 2$, *then the Agent's best response is (1) to choose* $(a_1 = 1, a_2 = 1)$ *if* $e_i(1 - e_j)B - k \geq 0$ *for all* $i = 1, 2$, $j \neq i$, *(2) to choose* $(a_i = 1, a_j = 0)$ *if* $[0 \geq e_j(1 - e_i)B - k, e_i \geq e_j,$ *and* $e_i B - k \geq 0]$, *and (3) to choose* $(a_1 = 0, a_2 = 0)$ *if* $e_i B - k \leq 0$ *for all* $i = 1, 2$.

3. If the Principal retains the Agent if, and only if, $o_i = s$ *for specific* $i = 1, 2$, *then the Agent's best response is to choose* $(a_i = 1, a_j = 0)$ *if* $e_i B - k \geq 0$, *and to choose* $(a_i = 0, a_j = 0)$ *if* $e_i B - k \leq 0$.

4. If the Principal always or never retains the Agent, then the Agent's best response is to choose $(a_1 = 0, a_2 = 0)$.

Proof of Lemma A. 2. Suppose first that the Principal retains the Agent if, and only if, $(o_1 = s, o_2 = s)$. Let us denote this retention rule by r_s. Then

$$U_A((a_1 = 1, a_2 = 1), r_s) = e_1 e_2 B - 2k$$

$$U_A((a_i = 1, a_j = 0), r_s) = -k$$

$$U_A((a_1 = 0, a_2 = 0), r_s) = 0.$$

Choosing $(a_i = 1, a_j = 0)$ is never a best response because $-k < 0$. Hence, choosing $(a_1 = 1, a_2 = 1)$ is the Agent's best response to the strict retention rule if, and only if, $e_1 e_2 B - 2k \geq 0$, while choosing $(a_1 = 0, a_2 = 0)$ is the Agent's best response if, and only if, $e_1 e_2 B - 2k \leq 0$.

Suppose now that the Principal retains the Agent if, and only if, $o_i = s$ for at least some $i = 1, 2$. Let us denote this retention rule by r_m. Then

$$U_A((a_1 = 1, a_2 = 1), r_m) = (e_1 e_2 + e_1(1 - e_2) + (1 - e_1)e_2)B - 2k$$

$$U_A((a_1 = 1, a_j = 0), r_m) = e_1 B - k$$

$$U_A((a_1 = 0, a_2 = 1), r_m) = e_2 B - k$$

$$U_A((a_1 = 0, a_2 = 0), r_m) = 0.$$

Hence, choosing $(a_1 = 1, a_2 = 1)$ is the Agent's best response to the moderate retention rule if, and only if, the following two conditions are satisfied: (1) $(e_1e_2+e_1(1-e_2)+(1-e_1)e_2)B-2k \geq e_iB - k$ for all $i = 1, 2$, and (2) $(e_1e_2 + e_1(1 - e_2) + (1 - e_1)e_2)B - 2k \geq 0$. Condition (1), in turn, is satisfied if, and only if, $[e_1(1 - e_2)B - k \geq 0, \text{ and } (1 - e_1)e_2B - k \geq 0]$ which implies condition (2). Choosing $(a_i = 1, a_j = 0)$ is the Agent's best response if the following three conditions hold: (1) $e_iB-k \geq 0$, (2) $e_iB-k \geq e_jB-k$, and (3) $e_iB-k \geq (e_1e_2+e_1(1-e_2)+(1-e_1)e_2)B-2k$ which is equivalent to $[0 \geq e_j(1 - e_i)B - k, e_i \geq e_j, \text{ and } e_iB - k \geq 0]$. Finally, choosing $(a_i = 0, a_j = 0)$ is the Agent's best response if the following two conditions hold: (1) $0 \geq e_iB - k$ for all $i = 1, 2$, and (2) $0 \geq (e_1e_2 + e_1(1 - e_2) + (1 - e_1)e_2)B - 2k$. Because (1) implies (2), this is equivalent to $e_iB - k \leq 0$ for all $i = 1, 2$.

Suppose next that the Principal retains the Agent if, and only if, $o_i = s$ for a specific $i = 1, 2$. Denote this retention rule by r_i. Then

$$U_A((a_1 = 1, a_2 = 1), r_i) = e_i B - 2k$$

$$U_A((a_i = 1, a_j = 0), r_i) = e_i B - k$$

$$U_A((a_i = 0, a_j = 1), r_i) = -k$$

$$U_A((a_1 = 0, a_2 = 0), r_i) = 0.$$

Hence, choosing $(a_1 = 1, a_2 = 1)$ is never a best-response to the i^{th}-task retention rule because $e_iB - 2k < e_iB - k$. Similarly, $(a_i = 0, a_j = 1)$ is never a best response because $-k < 0$. Choosing $(a_i = 1, a_j = 0)$ is thus the Agent's best response to the i^{th}-task retention rule if, and only if, $e_iB - k \geq 0$, while choosing $(a_i = 0, a_j = 0)$ is the Agent's best response if, and only if, $e_iB - k \leq 0$.

If the Principal never retains (or always retains), then $U_A(a_1 = 1, a_2 = 1) < U_A(a_i = 1, a_j = 0) < U_A(a_1 = 0, a_2 = 0)$ and the Agent's best response is to choose $(a_1 = 0, a_2 = 0)$. □

Proof of Proposition 1*.* Follows from Lemmata A. 1 and A. 2 by looking for mutual best responses. □

Corollary A. 1. *From Proposition 1, we can give a formal statement of the conditions that define* $\mathcal{M}$ *and* $\mathcal{S}$*. We have,* $\mathcal{M} = \{x \in \mathcal{X} | 1 - \frac{k}{e_jB} \geq e_i \geq \frac{k}{(1-e_j)B}, e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$ *for all* $i = 1, 2\}$ *and* $\mathcal{S} = \{x \in \mathcal{X} | e_i \geq \frac{2k}{e_jB}, e_i^H(1 - e_j^H) \leq e_i^L(1 - e_j^L)$ *for all* $i = 1, 2\}$.

The following Lemma shows that the conditions on (e_1, e_2) and on $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ that define $\mathcal{M}$ and $\mathcal{S}$ can jointly be satisfied.

Lemma A. 3.

1. *For any* $(e_1, e_2) \in (0, 1)^2$ *there exists an infinity of* $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ *such that* $e_i = \pi e_i^H + (1 - \pi)e_i^L$ *and* $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$ *for all* $i, j = 1, 2, j \neq i$.

2. *For any* $(e_1, e_2) \in (0, 1)^2$ *there exists an infinity of* $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ *such that* $e_i = \pi e_i^H + (1 - \pi)e_i^L$ *and* $e_i^H(1 - e_j^H) \leq e_i^L(1 - e_j^L)$ *for all* $i, j = 1, 2, j \neq i$.

Proof of Lemma A. 3*.*

1. Suppose first $e_1 = e_2 \in (0, 1)$. Note that for any $e_1^H, e_1^L, e_2^H, e_2^L$ such that (1) $e_1^H = e_2^H$, (2) $e_1^L = e_2^L$, (3) $e_1^L < e_1 < e_1^H$, and (4) $|e_1^H - 1/2| \leq |e_1^L - 1/2|$, we have $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$ for all $i, j = 1, 2, j \neq i$. It is easy to see that there is an infinity of $e_1^H, e_1^L, e_2^H, e_2^L$ that satisfy conditions (1) through (4). Now let $\pi = \frac{e_i - e_i^L}{e_i^H - e_i^L}$. Because $e_i^H > e_i > e_i^L$, we have $\pi \in (0, 1)$. Moreover, some simple algebra

establishes that $e_i = \pi e_i^H + (1-\pi)e_i^L$. WLOG suppose next that $1 > e_1 > e_2 > 0$. Some simple algebra establishes that $e_i^H(1-e_j^H) > e_i^L(1-e_j^L)$ for all $i, j = 1, 2, j \neq i$ if, and only if $e_1^H < 1 - e_2^L$ and $e_1^L < 1 - e_2^H$. Choose any e_1^H and e_2^L such that (1) $e_1^H < 1 - e_2^L$, (2) $e_2 > e_2^L > 0$, and (3) $\frac{1-e_2^L}{e_2-e_2^L}e_1 > e_1^H > e_1$. It is easy to see that there is an infinity of e_1^H's and e_2^L's that satisfy conditions (1) through (3). Now choose $\pi \in (\frac{e_2-e_2^L}{1-e_2^L}, \frac{e_1}{e_1^H}) \subset (0,1)$ close enough to $\frac{e_1}{e_1^H}$. By condition (3) we have $\frac{e_2-e_2^L}{1-e_2^L} < \frac{e_1}{e_1^H}$. Therefore such a value of π exists. Now let $e_1^L = \frac{e_1-\pi e_1^H}{1-\pi}$ and $e_2^H = \frac{e_2-(1-\pi)e_2^L}{\pi}$. By this definition of e_1^L and e_2^H, we have $e_i = \pi e_i^H + (1-\pi)e_i^L$ for all $i = 1, 2$. Moreover, because $\pi < \frac{e_1}{e_1^H}$, we have $e_1^L > 0$. Similarly, $\pi > \frac{e_2-e_2^L}{1-e_2^L}$ implies that $1 > e_2^H$. Finally, we have $e_2^H < 1 - e_1^L$ if, and only if, $\frac{e_2-(1-\pi)e_2^L}{\pi} < 1 - \frac{e_1-\pi e_1^H}{1-\pi}$, if, and only if, $f(\pi) := \pi^2(1-e_2^L-e_1^H) + \pi(-1-e_2+2e_1^L+e_1) + e_2 - e_2^L < 0$. As $e_1^H < 1$, which by assumption is lower than $\frac{e_1}{e_2}$, some simple algebra shows that $f(\frac{e_1}{e_1^H}) < 0$. As $f(\pi)$ is a continuous function of π, we have $f(\pi) < 0$ for any value of π close enough to $\frac{e_1}{e_1^H}$, which establishes the result.

2. Proceeding in a similar way than just above yields the result.

□

2 Comparing Institutions (Proof of Proposition 2)

Proposition 2. *1. If effort is positive and (weakly) higher under bundling, selection is strictly better under bundling.*

2. If effort is higher under unbundling, selection may be better under bundling or unbundling, depending on the values of e_i^H, e_i^L, $i = 1, 2$.

Proof of Proposition 2. Denote by $U_P^U(a_1, a_2)$ the ex ante post-election welfare of the

Principal under unbundling when (a_1, a_2) is exerted in equilibrium. We have

$$
\begin{aligned}
U_P^U(a_1 = 1, a_2 = 1) &= \sum_{i=1}^{2} \pi \left[e_i^H R + (1 - e_i^H) \pi R \right] + (1 - \pi)(1 - e_i^L) \pi R \\
U_P^U(a_i = 1, a_j = 0) &= \pi \left[e_i^H R + (1 - e_i^H) \pi R \right] + (1 - \pi)(1 - e_i^L) \pi R + \pi R \\
U_P^U(a_1 = 0, a_2 = 0) &= 2\pi R.
\end{aligned}
$$

Simple algebra establishes that $U_P^U(a_1 = 1, a_2 = 1) > U_P^U(a_i = 1, a_j = 0) > U_P^U(a_1 = 0, a_2 = 0)$.

Similarly, denote $U_P^B((a_1, a_2), r)$ the ex ante post-election welfare of the Principal under bundling when (a_1, a_2) is exerted by the Agent and the Principal uses retention rule r.

We have

$$
\begin{aligned}
U_P^B(a_1 = 1, a_2 = 1, r_s) &= \pi \left[e_1^H e_2^H 2R + (1 - e_1^H e_2^H) \pi 2R \right] + (1 - \pi)(1 - e_1^L e_2^L) \pi 2R \\
U_P^B(a_1 = 1, a_2 = 1, r_m) &= \pi \left[(e_1^H + e_2^H - e_1^H e_2^H) 2R + (1 - e_1^H)(1 - e_2^H) \pi 2R \right] \\
&\quad + (1 - \pi)(1 - e_1^L)(1 - e_2^L) \pi 2R \\
U_P^B(a_i = 1, a_j = 0, r_i) &= \pi \left[e_i^H 2R + (1 - e_i^H) \pi 2R \right] + (1 - \pi)(1 - e_i^L) \pi 2R \\
U_P^U(a_1 = 0, a_2 = 0, r) &= 2\pi R.
\end{aligned}
$$

Simple algebra establishes (1) that $U_P^B(a_i = 1, a_j = 0, r_i) > U_P^B(a_1 = 0, a_2 = 0, r)$, (2) that $U_P^B(a_1 = 1, a_2 = 1, r_m) > U_P^B(a_i = 1, a_j = 0, r_i)$ when $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$ for all $i = 1, 2, j \neq i$, and (3) that $U_P^B(a_1 = 1, a_2 = 1, r_s) > U_P^B(a_i = 1, a_j = 0, r_i)$ when $e_i^H(1 - e_j^H) \leq e_i^L(1 - e_j^L)$ for all $i = 1, 2, j \neq i$.

Some more algebra then establishes (1) that $U_P^B(a_1 = 1, a_2 = 1, r_m) > U_P^U(a_1 = 1, a_2 = 1)$ when $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$ for all $i = 1, 2, j \neq i$, (2) $U_P^B(a_1 = 1, a_2 = 1, r_s) > U_P^U(a_1 = 1, a_2 = 1)$ when $e_i^H(1 - e_j^H) \leq e_i^L(1 - e_j^L)$ for all $i = 1, 2, j \neq i$, and (3) that $U_P^B(a_i = 1, a_j =$

$0, r_i) > U_P^U(a_i = 1, a_j = 0)$, which, combined with $U_P^U(a_1 = 1, a_2 = 1) > U_P^U(a_i = 1, a_j = 0) > U_P^U(a_1 = 0, a_2 = 0)$, establishes Part 1 of Proposition 2.

Finally, we have (1) $U_P^U(a_1 = 1, a_2 = 1) > U_P^B(a_i = 1, a_j = 0, r_i)$ if, and only if, $e_i^H - e_i^L < e_j^H - e_j^L$, (2) $U_P^U(a_1 = 1, a_2 = 1) > U_P^B(a_1 = 0, a_2 = 0, r)$, and (3) $U_P^U(a_i = 1, a_j = 0) > U_P^B(a_1 = 0, a_2 = 0, r) = U_P^U(a_1 = 0, a_2 = 0)$, which establishes Part 2 of Proposition 2. □

3 Comparing Institutions cont'd (Proof of Proposition 3)

Proposition 3. *1. There exists $M \subset \mathcal{M}$, $M \neq \emptyset$, such that $M \cap \mathcal{U} = \emptyset$ and bundling has a strict incentive advantage over unbundling if, and only if, $x \in M$.*

2. There exists $U \subset \mathcal{U}$, $U \neq \emptyset$ such that $U \cap \mathcal{M} = \emptyset$, $U \cap \mathcal{S} = \emptyset$, and unbundling has a strict incentive advantage over bundling if, and only if, $x \in U$.

Proof of Proposition 3. We prove the following two statements which entail Proposition 3:

1. Effort on both tasks can be sustained in equilibrium under bundling, but not under unbundling, if, and only if, for all $i = 1, 2, j \neq i$, (1) $\frac{k}{(1-e_j)B} \leq e_i \leq \min\{\frac{2k}{e_jB}, 1 - \frac{k}{e_jB}\}$, and (2) $e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L)$. There exists an infinity of vectors $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ that satisfy conditions (1) and (2) if, and only if, $B > 4k$.

2. If $4k \geq B > 2k$, then effort on both tasks can be sustained in equilibrium under unbundling, but not under bundling, if $\frac{2k}{e_2B} > e_1 \geq \frac{e_2k}{e_2B-k}$. If $B > 4k$, then effort on both tasks can be sustained in equilibrium under unbundling, but not under bundling, if $\frac{2k}{e_2B} > e_1 \geq \frac{e_2k}{e_2B-k}$ and if $e_1 \notin \left[\frac{k}{(1-e_2)B}, 1 - \frac{k}{e_2B}\right]$.

We first establish that for all parameter values such that there exists a strict incentives equilibrium under bundling, there exists a feasible pair (B_1, B_2) such that both Agents exert effort under unbundling. Consequently, for any $x \in \mathcal{X}$ such that, in equilibrium, effort is exerted on both tasks under bundling, yet not under unbundling, it must be the case that the Principal uses the moderate retention rule. If the Principal uses the strict retention rule, the Agent chooses $(a_1 = 1, a_2 = 1)$ if, and only if, $e_1 e_2 B - 2k \geq 0$ or equivalently $1 \geq e_1 \geq \frac{2k}{e_2 B}$. Similarly, there exists (B_1, B_2) such that both Agents exert effort under unbundling if, and only if, $1 \geq e_1 \geq \frac{e_2 k}{e_2 B - k}$. We now prove that for all $(e_1, e_2) \in [0,1]^2$, if $1 \geq e_1 \geq \frac{2k}{e_2 B}$ then $1 \geq e_1 \geq \frac{e_2 k}{e_2 B - k}$. Note first that if $0 \leq e_2 < \frac{2k}{B}$, then $\frac{2k}{e_2 B} > 1$ and there is no value of e_1 that satisfies $1 \geq e_1 \geq \frac{2k}{e_2 B}$. Similarly, if $0 < B < 2k$ there is no $(e_1, e_2) \in [0,1]^2$ that satisfies $1 \geq e_1 \geq \frac{2k}{e_2 B}$. We have $\frac{e_2 k}{e_2 B - k} < \frac{2k}{e_2 B}$ if, and only if, $Q(e_2) := e_2^2 B - 2e_2 B + 2k < 0$. If $e_2 = \frac{2k}{B}$, then $Q(e_2) \leq 0$ whenever $B \geq 2k$. Moreover, $\frac{dQ}{de_2} = 2e_2 B - 2B < 0$ for all $e_2 \in [0,1)$. Hence, $\frac{e_2 k}{e_2 B - k} \leq \frac{2k}{e_2 B}$ for all $e_2 \in \left[\frac{2k}{B}, 1\right]$, which establishes the claim.

We now show that if $B > 4k$ there exists a non-empty open set of vectors $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ for which $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under bundling, but not under unbundling. Let e^u be the intersection between $e_1 = \frac{e_2 k}{e_2 B - k}$ and $e_1 = e_2$. We have $e^u = \frac{2k}{B}$. Note that $e^u < 1$ as $B > 4k$. For the rest of the paragraph we restrict attention to values of (e_1, e_2) such that $e_1 = e_2$. Abusing notation slightly, we work with $e = e_1 = e_2$. For any $e < e^u$, effort is exerted on at most one task under unbundling because we have $e < \frac{ek}{eB - k}$ whenever $e < e^u$. We now derive the set of values of e that satisfy $e(1-e)B - k \geq 0$. We have $e(1-e)B - k = 0$ for $\frac{1}{2} - \frac{\sqrt{B(B-4k)}}{2B} := \underline{e}^m$ and for $\frac{1}{2} + \frac{\sqrt{B(B-4k)}}{2B} := \overline{e}^m$. If $B > 4k$, $\underline{e}^m$ and $\overline{e}^m$ are well-defined and $0 \leq \underline{e}^m < 1/2 < \overline{e}^m \leq 1$. Moreover, $e(1-e)B - k$ reaches its maximum at $e = 1/2$ and is strictly increasing on $[0, 1/2)$ and strictly decreasing on $(1/2, 1]$. Thus, for all $e \in [\underline{e}^m, \overline{e}^m]$, we have $e(1-e)B - k \geq 0$. Simple algebra shows that $\underline{e}^m < e^u$ if $B > 4k$. It follows that for all $e \in [\underline{e}^m, e^u)$, $(a_1 = 1, a_2 = 1)$ cannot be sustained under unbundling, but can be sustained under bundling, provided the Principal uses the moderate retention rule.

In equilibrium, the Principal uses the moderate retention rule if, and only if, $e_i^H(1-e_j^H) \geq e_i^L(1-e_j^L)$ for all $i, j = 1, 2, j \neq i$. Lemma A. 3 shows that there exists for any $(e_1, e_2) \in (0,1)^2$ an infinity of $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ such that $e_i = \pi e_i^H + (1-\pi)e_i^L$ and $e_i^H(1-e_j^H) \geq e_i^L(1-e_j^L)$ for all $i, j = 1, 2, j \neq i$.

We now show that if $B > 4k$, there exists $(e_1, e_2) \in [0,1]^2$ such that $\frac{2k}{e_2 B} > e_1 \geq \frac{e_2 k}{e_2 B - k}$ and $e_1 \notin \left[\frac{k}{(1-e_2)B}, 1 - \frac{k}{e_2 B}\right]$. Let $e_2 = \frac{2k}{B}$. Then $\frac{2k}{e_2 B} = 1$, $1 - \frac{k}{e_2 B} = 1/2$, $\frac{k}{(1-e_2)B} = \frac{k}{B-2k}$, and $\frac{e_2 k}{e_2 B - k} = \frac{2k}{B}$. Because $B > 4k$, we have $\frac{k}{(1-e_2)B} = \frac{k}{B-2k} < 1/2$, and $\frac{e_2 k}{e_2 B - k} = \frac{2k}{B} < 1/2$. Hence, if $e_2 = \frac{2k}{B}$, then any $e_1 \in (1/2, 1)$ satisfies the required conditions. Moreover, it is easy to see that around any (e_1, e_2) such that $e_1 \in (1/2, 1)$ and $e_2 = \frac{2k}{B}$ there is an open ball that satisfies the required conditions. □

4 Transparency of Actions (Proof of Proposition 4)

Proposition 4. *1.* $\mathcal{M} = \mathcal{M}^T$.

2. While transparency never decreases and may increase the strict incentive advantage of unbundling, if bundling has a strict incentive advantage with no transparency (i.e. under the moderate retention rule) it will also with transparency.

To prove Proposition 4, we first provide, in Proposition A. 1, a characterization of the Agent's level of effort under bundling with transparency of actions.

Proposition A. 1. *On the equilibrium path of play under bundling with transparency of actions:*

1. *The Agent chooses to exert effort on both tasks if, and only if, the complexity of each task is moderate,* $1 - \frac{k}{e_j B} \geq e_i \geq \frac{k}{(1-e_j)B}$, *and the Principal's estimation of the Agent's*

competence increases when the outcome is success on at least one task, $e_i^H(1-e_j^H) \geq e_i^L(1-e_j^L)$ *for all* $i = 1, 2$.

2. *The Agent chooses to exert effort only on task* i *if, and only if, all of the following three conditions hold: (1) the complexity of that task is sufficiently low,* $e_i \geq k/B$, *(2) it is (weakly) lower than the complexity of task* $j \neq i$, $e_i \geq e_j$, *and (3) the conditions to sustain an equilibrium in which the Agent chooses to exert effort on both tasks (stated in part 1.) are not satisfied.*

3. *The Agent chooses to exert no effort on either task if, and only if, the complexity of each task is sufficiently high:* $e_i \leq k/B$ *for all* $i = 1, 2$.

Proof of Proposition A. 1. Suppose the Agent chooses $(a_i = 1, a_j = 0)$. By Lemma A. 1, the Principal then retains the Agent if, and only if, $o_i = s$. Consequently, the Agent's expected utility from choosing $(a_i = 1, a_j = 0)$ is $e_i B - k$.

Suppose next the Agent chooses $(a_1 = 1, a_2 = 1)$. By Lemma A. 1, the Principal then uses one of the following three retention rules:

1. If $e_i^H(1-e_j^H) < e_i^L(1-e_j^L)$ for all $i = 1, 2$, the Principal retains if, and only if $(o_1 = s, o_2 = s)$.

2. If $e_i^H(1-e_j^H) > e_i^L(1-e_j^L)$ for all $i = 1, 2$, the Principal retains if, and only if $o_i = s$ for at least some task $i = 1, 2$.

3. If $e_i^H(1-e_j^H) > e_i^L(1-e_j^L)$ for $i = 1, 2$, and $e_j^H(1-e_i^H) < e_j^L(1-e_i^L)$ for $j \neq i$, the Principal retains if, and only if $o_i = s$ for $i = 1, 2$.

In case 1. the Agent's expected utility from choosing $(a_1 = 1, a_2 = 1)$ is $e_1 e_2 B - 2k$. In case 2. the Agent's expected utility from choosing $(a_1 = 1, a_2 = 1)$ is $(e_1 + e_2 - e_1 e_2)B - 2k$. In case 3. the Agent's expected utility from choosing $(a_1 = 1, a_2 = 1)$ is $e_i B - 2k$. Finally choosing $(a_1 = 0, a_2 = 0)$ induces, by Lemma A. 1 again, the Principal to dismiss the Agent who then receives a payoff of 0.

The Agent then chooses the effort allocation (a_1, a_2) which maximizes his expected utility given the retention rule that is induced by (a_1, a_2). Because $e_1 e_2 B - 2k < e_i B - k$ and $e_i B - 2k < e_i B - k$ for all $i = 1, 2$, the Agent never chooses $(a_1 = 1, a_2 = 1)$ if it induces retention rule 1. or 3. The Agent chooses $(a_1 = 1, a_2 = 1)$ if, and only if, $(a_1 = 1, a_2 = 1)$ induces retention rule 2. and $(e_1 + e_2 - e_1 e_2)B - 2k \geq e_i B - k \geq 0$ for all $i = 1, 2$, which is equivalent to $e_i(1 - e_j)B - k \geq 0$ for all $i = 1, 2$. The Agent chooses $(a_i = 1, a_j = 0)$ if, and only if, one of the two following sets of conditions hold: (1) $(a_1 = 1, a_2 = 1)$ induces retention rule 2. and $e_i B - k \geq \max\{e_j B - k, (e_1 + e_2 - e_1 e_2)B - 2k, 0\}$, or (2) $(a_1 = 1, a_2 = 1)$ does not induce retention rule 2. and $e_i B - k \geq \max\{e_j B - k, 0\}$. The Agent chooses $(a_1 = 0, a_2 = 0)$ if, and only if, $0 > e_i B - k$ for all $i = 1, 2$. Note that $0 > e_i B - k$ for all $i = 1, 2$, implies $0 > (e_1 + e_2 - e_1 e_2)B - 2k$. □

Proof of Proposition 4. By corollary A. 1, we have $\mathcal{M} = \{x \in \mathcal{X} | 1 - \frac{k}{e_j B} \geq e_i \geq \frac{k}{(1-e_j)B}, e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L) \text{ for all } i = 1, 2\}$. By Part 1 of Proposition A. 1, we have $\mathcal{M}^T = \{x \in \mathcal{X} | 1 - \frac{k}{e_j B} \geq e_i \geq \frac{k}{(1-e_j)B}, e_i^H(1 - e_j^H) \geq e_i^L(1 - e_j^L) \text{ for all } i = 1, 2\}$. It follows that $\mathcal{M} = \mathcal{M}^T$. Part 2 follows directly from propositions 3 and A. 1. □

5 Influence of Special Interests (Proof of Proposition 5)

Proposition 5. *When the presence of IG makes a difference for the Agent's choice,*

1. *it can create a strict incentive advantage of bundling over unbundling and vice versa;*

2. *if it creates a strict incentive advantage of bundling, it is for $x \in \mathcal{M}$.*

To prove Proposition 5, we proceed in the following steps. We first characterize, in Proposition A. 2, the equilibrium levels of effort of the Agent under bundling in the presence of IG, using the bribe levels described in Lemma A. 4 as a stepping stone. We then characterize the equilibrium levels of effort of the Agents under unbundling in the presence of IG in Proposition A. 3. Using these results, we finally prove Proposition 5.

Lemma A. 4. *Under bundling:*

1. *Suppose the Principal retains the Agent if, and only if, there is success on task 1 and assume that $e_1 B - k \geq 0$. Then, IG offers the Agent a bribe $b = e_1 B - k$ if, and only if, $u_{IG} \geq B - k/e_1$.*

2. *Suppose the Principal retains the Agent if, and only if, there is success on both tasks and assume that $e_1 e_2 B - 2k \geq 0$. Then, IG offers the Agent a bribe $b = e_1 e_2 B - 2k$ if, and only if, $u_{IG} \geq e_2 B - 2k/e_1$.*

3. *Suppose the Principal retains the Agent if, and only if, there is success on at least one task and assume that $e_i(1 - e_j)B - k \geq 0$ for all $i = 1, 2$. Then, IG offers the Agent a bribe $b = e_1(1 - e_2)B - k$ if, and only if, $u_{IG} \geq (1 - e_2)B - k/e_1 =: \check{u}(e_1, e_2)$.*

Proof of lemma A. 4. 1. Suppose the Principal retains the Agent if, and only if, there is success on task 1 and assume that $e_1 B - k \geq 0$. Then, if IG does not offer a bribe, the Agent chooses $(a_1 = 1, a_2 = 0)$. The Agent's expected utility is then $e_1 B - k$. If IG offers the Agent a bribe b and the Agent accepts the bribe, the Agent chooses $(a_1 = 0, a_2 = 0)$ and receives an expected utility of b. It follows that the Agent accepts the bribe b if, and only if, $b \geq e_1 B - k$. IG thus chooses between the lowest bribe that the Agent accepts, i.e. $b = e_1 B - k$ and $b = 0$. Upon offering $b = e_1 B - k$, IG receives a payoff of $u_{IG} - b = u_{IG} - (e_1 B - k)$. Upon offering $b = 0$, IG receives a payoff of $(1 - e_1)u_{IG}$. Hence, IG offers $b = e_1 B - k$ if, and only if, $u_{IG} \geq B - k/e_1$.

2. Suppose the Principal retains the Agent if, and only if, there is success on both tasks and assume that $e_1e_2B - 2k \geq 0$. Then, if IG does not offer a bribe, the Agent chooses $(a_1 = 1, a_2 = 1)$. The Agent's expected utility is then $e_1e_2B - 2k$. If IG offers the Agent a bribe b and the Agent accepts the bribe, the Agent chooses $(a_1 = 0, a_2 = 0)$ and receives an expected utility of b. It follows that the Agent accepts the bribe b if, and only if, $b \geq e_1e_2B - 2k$. IG thus chooses between the lowest bribe that the Agent accepts, i.e. $b = e_1e_2B - 2k$ and $b = 0$. Upon offering $b = e_1e_2B - 2k$, IG receives a payoff of $u_{IG} - b = u_{IG} - (e_1e_2B - 2k)$. Upon offering $b = 0$, IG receives a payoff of $(1 - e_1)u_{IG}$. Hence, IG offers $b = e_1e_2B - 2k$ if, and only if, $u_{IG} \geq e_2B - 2k/e_1$.

3. Suppose the Principal retains the Agent if, and only if, there is success on at least one task and assume that $e_i(1 - e_j)B - k \geq 0$ for all $i = 1, 2, j \neq i$. Then, if IG does not offer a bribe, the Agent chooses $(a_1 = 1, a_2 = 1)$. The Agent's expected utility is then $(e_1e_2 + e_1(1 - e_2) + (1 - e_1)e_2)B - 2k$. Note that $e_2(1 - e_1)B - k \geq 0$ implies $e_2B - k \geq 0$. Hence, if IG offers the Agent a bribe b and the Agent accepts the bribe, the Agent chooses $(a_1 = 0, a_2 = 1)$ and receives an expected utility of $b + e_2B - k$. It follows that the Agent accepts the bribe b if, and only if, $b + e_2B - k \geq (e_1e_2 + e_1(1 - e_2) + (1 - e_1)e_2)B - 2k$, i.e. if, and only if, $b \geq e_1(1 - e_2)B - k$. IG thus chooses between the lowest bribe that the Agent accepts, i.e. $b = e_1(1 - e_2)B - k$ and $b = 0$. Upon offering $b = e_1(1 - e_2)B - k$, IG receives a payoff of $u_{IG} - b = u_{IG} - (e_1(1 - e_2)B - k)$. Upon offering $b = 0$, IG receives a payoff of $(1 - e_1)u_{IG}$. Hence, IG offers $b = e_1(1 - e_2)B - k$ if, and only if, $u_{IG} \geq (1 - e_2)B - k/e_1$.

□

Proposition A. 2. *On the equilibrium path of play under bundling with IG:*

1. *The Agent chooses* $(a_1 = 1, a_2 = 1)$ *if, and only if, either*

   (a) the complexity of each task, as well as the value of failure to IG, are sufficiently low,

$e_1 \geq \frac{2k}{e_2 B - u_{IG}}$ *and the Principal's estimation of the Agent's competence decreases unless the outcome is success on both tasks,* $e_i^H(1-e_j^H) \leq e_i^L(1-e_j^L)$ *for all* $i = 1, 2, j \neq i$*; or*

(b) *the complexity of each task is moderate and the value of failure to IG is sufficiently low,* $1 - \frac{k}{e_2 B} \geq e_i \geq \frac{k}{(1-e_2)B - u_{IG}}$*, and the Principal's estimation of the Agent's competence increases when the outcome is success on at least one task,* $e_1^H(1 - e_2^H) \geq e_1^L(1 - e_2^L)$*, and* $e_2^H(1 - e_1^H F(\check{u}(e_1, e_2))) \geq e_2^L(1 - e_1^L F(\check{u}(e_1, e_2)))$.

In case (a), the Principal adopts the strict retention rule, in case (b), the moderate retention rule.

2. *The Agent chooses* $(a_1 = 1, a_2 = 0)$ *when the complexity on task 1, as well as the value of failure to IG, are sufficiently low,* $e_1 \geq k/(B - u_{IG})$*, and the Principal adopts the* 1^{th}*-task retention rule.*

3. *The Agent chooses* $(a_1 = 0, a_2 = 1)$ *when the complexity on task 2 is sufficiently low,* $e_2 > k/B$*, and the Principal retains if* $(o_1 = f, o_2 = s)$ *and dismisses if* $(o_1 = f, o_2 = f)$.

4. *The Agent chooses* $(a_1 = 0, a_2 = 0)$ *if*

   (a) $e_1 < k/(B - u_{IG})$*, and* $e_2 < k/B$*, independent of the Principal's retention rule, or*

   (b) $e_1 < \frac{2k}{e_2 B - u_{IG}}$ *and the Principal adopts the strict retention rule (which requires* $e_i^H(1 - e_j^H) \leq e_i^L(1 - e_j^L)$ *for all* $i = 1, 2$*).*

Proof of Proposition A. 2. 1. From Proposition 1 we know that the Agent chooses to exert effort on both tasks if, and only if, the Principal uses either the strict retention rule or the moderate retention rule.

(a) Suppose the Principal uses the strict retention rule. The Agent then chooses to exert effort on both tasks if, and only if, $e_1 e_2 B - 2k \geq 0$ and IG did not bribe the Agent. From Lemma A. 4, IG, in turn, does not bribe the Agent if, and only if, $u_{IG} \leq e_2 B - 2k/e_1$ which is equivalent to $e_1 \geq \frac{2k}{e_2 B - u_{IG}} \geq \frac{2k}{e_2 B}$. The conditions under which it is sequentially rational for the Principal to use the strict retention rule are not altered by the possibility of IG influence. The derivation is similar to the one found in the proof of Lemma A. 1.

(b) Suppose the Principal uses the moderate retention rule. The Agent then chooses to exert effort on both tasks if, and only if, $1 - \frac{k}{e_2 B} \geq e_1 \geq \frac{k}{(1-e_2)B}$, and IG did not bribe the Agent. From Lemma A. 4, IG, in turn, does not bribe the Agent if, and only if, $u_{IG} \leq (1 - e_2)B - k/e_1 = \check{u}(e_1, e_2)$ which is equivalent to $e_1 \geq \frac{k}{(1-e_2)B - u_{IG}} \geq \frac{k}{(1-e_2)B}$. When IG bribes the Agent, the Agent chooses $(a_1 = 0, a_2 = 1)$ if, and only if, $e_2 B - k \geq 0$. Note that $1 - \frac{k}{e_2 B} \geq e_1 \geq \frac{k}{(1-e_2)B}$ implies $e_2 B - k \geq 0$.

We now derive the conditions under which it is sequentially rational for the Principal to use the moderate retention rule given that the Principal believes that the Agent and IG are best-responding to such a retention strategy. To understand the construction of the beliefs, remember that the Principal is uncertain about the value u_{IG} that IG attaches to policy failure and that u_{IG} is drawn from a distribution function $F(\cdot)$ with full support on the non-negative real line $\mathbb{R}_+$. It follows that the Principal expects the following action profile: with probability $F(\check{u}(e_1, e_2)) \in (0, 1)$ IG does not to bribe the Agent who chooses $(a_1 = 1, a_2 = 1)$, while with probability $(1 - F(\check{u}(e_1, e_2)))$ IG bribes the Agent who then chooses $(a_1 = 0, a_2 = 1)$. We denote $(\mathring{a}_1^F, \mathring{a}_2 = 1)$ these expectations of the Principal about the Agent's actions. We thus have

$$
\begin{aligned}
Pr(\theta &= \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) = \frac{F(\check{u}(e_1,e_2))e_1^H e_2^H \pi}{F(\check{u}(e_1,e_2))e_1^H e_2^H \pi + F(\check{u}(e_1,e_2))e_1^L e_2^L \pi}, \\
Pr(\theta &= \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \\
&= \frac{\left[(1-e_2^H)(1-e_1^H F(\check{u}(e_1,e_2)))\right]\pi}{\left[(1-e_2^H)(1-e_1^H F(\check{u}(e_1,e_2)))\right]\pi + \left[(1-e_2^L)(1-e_1^L F(\check{u}(e_1,e_2)))\right](1-\pi)}, \\
Pr(\theta &= \theta_H|(o_1 = s, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \\
&= \frac{F(\check{u}(e_1,e_2))e_1^H(1-e_2^H)\pi}{F(\check{u}(e_1,e_2))e_1^H(1-e_2^H)\pi + F(\check{u}(e_1,e_2))e_1^L(1-e_2^L)\pi}, \\
Pr(\theta &= \theta_H|(o_1 = f, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \\
&= \frac{\left[e_2^H - F(\check{u}(e_1,e_2))e_1^H e_2^H\right]\pi}{\left[e_2^H - F(\check{u}(e_1,e_2))e_1^H e_2^H\right]\pi + \left[e_2^L - F(\check{u}(e_1,e_2))e_1^L e_2^L\right](1-\pi)}.
\end{aligned}
$$

As in the baseline model, the Principal updates favorably on the type of the Agent upon observing success on both tasks, and negatively upon observing failure on both tasks. Indeed, we have $Pr(\theta = \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) > \pi$ and $Pr(\theta = \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) < \pi$, because $e_i^H > e_i^L$ for all $i = 1, 2$. But, the conditions under which the Principal updates favorably upon observing success on one task and failure on another differ from those of the baseline model. While, as in the baseline model, $Pr(\theta = \theta_H|(o_1 = s, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \geq \pi$ if, and only if, $e_1^H(1-e_2^H) \geq e_1^L(1-e_2^L)$, we have $Pr(\theta = \theta_H|(o_1 = f, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \geq \pi$ if, and only if, $e_2^H(1 - e_1^H F(\check{u}(e_1, e_2))) \geq e_1^L(1 - e_2^L F(\check{u}(e_1, e_2)))$.

2. Suppose the Principal retains if, and only if, $o_1 = s$. The Agent then chooses $(a_1 = 1, a_2 = 0)$ if, and only if, $e_1 B - k \geq 0$ and IG did not bribe the Agent. From Lemma A. 4, IG, in turn, does not bribe the Agent if, and only if, $u_{IG} \leq B - k/e_1$ which is equivalent to $e_1 \geq \frac{k}{B-u_{IG}} \geq \frac{k}{B}$. The conditions under which it is sequentially rational for the Principal to choose to retain if, and only if, $o_1 = s$ are not altered by the possibility of IG influence. The derivation is similar to the one found in the proof of Lemma A. 1.

3. Follows directly from Lemmata A. 1 and A. 2.

4. (a) Follows directly from Lemmata A. 1 and A. 2 and the previous steps in the present proof.

   (b) Suppose the Principal retains the Agent if, and only if, $(o_1 = s, o_2 = s)$, which requires $e_i^H(1 - e_j^H) \leq e_i^L(1 - e_j^L)$ for all $i = 1, 2$, and suppose that $e_1 < \frac{2k}{e_2 B - u_{IG}}$. Then IG bribes the Agent who, consequently, does not exert effort on task 1. But then, as the Principal retains if, and only if, $(o_1 = s, o_2 = s)$, the expected utility to the Agent of exerting effort on task 2 is $-k$. Hence, the Agent does not exert any effort at all.

□

Proposition A. 3. *For all* (e_i, B_i, k) *such that* $e_i B_i - k \geq 0$,

1. *the Interest Group offers a bribe* $b_u = e_1 B_1 - k$ *if, and only if,* $u_{IG} \geq B_1 - k/e_1 \geq 0$;

2. *Agent* A_2 *chooses to exert effort, and Agent* A_1 *accepts the bribe if, and only if,* $b \geq e_1 B_1 - k$, *and chooses* $a_1 = 0$ *if accepting and* $a_1 = 1$ *if rejecting; and*

3. *the Principal retains Agent* A_i *if, and only if, the outcome is success on task* i.

For all (e_i, B_i, k) *such that* $e_i B_i - k < 0$, *Agent* A_i *chooses to exert no effort on task* i *independent of the bribe and the Principal's retention rule.*

Proof of Proposition A. 3. Suppose the Principal reelects A_i if, and only if, $o_i = s$. The expected payoff to Agent A_i of choosing to exert effort is then $e_i B_i - k$. Thus, if $e_1 B_1 - k < 0$, Agent A_1 will choose not to exert effort independent of the offer from IG, which, therefore, has no incentive to influence A_1's behavior. If, on the other hand, $e_1 B_1 - k \geq 0$, A_1 exerts effort absent a bribe and IG has an incentive to influence him. In this case, A_1 accepts a bribe if, and only if, $b \geq e_1 B_1 - k$. If IG offers A_1 b_u, IG receives a payoff of $u_{IG} - b_u = u_{IG} - e_1 B_1 + k$.

If IG offers A_1 $b = 0$, IG receives an expected payoff of $(1-e_1)u_{IG}$. Hence, IG offers b_u if, and only if, $u_{IG} \geq B_1 - k/e_1$. Moreover, A_2 chooses $a_2 = 1$ if, and only if, $e_2B_2 - k \geq 0$. It is obvious that the Principal's retention rule is sequentially rational whenever $e_iB_i - k \geq 0$. $\square$

Proof of Proposition 5. We first show that for any $x \in \mathcal{S}$, if the presence of IG makes a difference for the Agent(s)'s choices, then it creates a strict incentive advantage for unbundling over bundling and never creates a strict incentive advantage for bundling. In a second step, we show that if $x \in \mathcal{M}$, then the presence of IG can create a strict incentive advantage for bundling over unbundling.

1. Following from Proposition A. 3, we know that if $e_1 \geq \frac{e_2k}{e_2(B-u_{IG})-k}$, there exists (B_1, B_2) such that effort on both tasks can be sustained under unbundling. We now show that if $B > 2k + u_{IG}$, we have $\frac{2k}{e_2B-u_{IG}} > \frac{e_2k}{e_2(B-u_{IG})-k}$ for all $e_2 \in \left[\frac{2k+u_{IG}}{B}, 1\right]$. Note that $\frac{2k}{e_2B-u_{IG}} > \frac{e_2k}{e_2(B-u_{IG})-k}$ if, and only if, $Q(e_2) := e_2^2B - e_2(2B - u_{IG}) + 2k < 0$. $\frac{dQ}{de_2} = 2e_2B - (2B - u_{IG})$. Hence, $Q(e_2)$ is decreasing on $\left[0, 1 - \frac{u_{IG}}{2B}\right]$ and increasing on $\left[1 - \frac{u_{IG}}{2B}, 1\right]$. It follows that the maximum of $Q(e_2)$ on $\left[\frac{2k+u_IG}{B}, 1\right]$ is reached at $e_2 = \frac{2k+u_{IG}}{B}$ or at $e_2 = 1$. Simple algebra shows that, if $B > 2k + u_{IG}$, then $Q(e_2 = \frac{2k+u_{IG}}{B}), Q(e_2 = 1) < 0$. It follows that there does not exist $(e_1^H, e_1^L, e_2^H, e_2^L, \pi, k, B, u_{IG})$ such that effort is exerted on both tasks under bundling and strict incentives, yet effort is not exerted on both tasks under unbundling. In other words, for all $x \in \mathcal{S}$, and for any $u_{IG} \geq 0$, unbundling has a weak incentive advantage over bundling in the presence of IG.

   Now choose any $x \in \mathcal{S}$. This requires, in particular, that $1 \geq e_1 \geq \frac{2k}{e_2B}$. Note that we established in the proof of Proposition 3 that $\frac{2k}{e_2B} > \frac{e_2k}{e_2B-k}$ for all $e_2 \in \left[\frac{2k}{B}, 1\right]$. Hence, there exists (B_1, B_2) such that effort is exerted on both tasks under unbundling for any $x \in \mathcal{S}$. It is easy to see that there exist $\underline{u}_{IG}(e_1, e_2), \overline{u}_{IG}(e_1, e_2) \in [0, e_2B)$ such that $1 \geq e_1 = \frac{2k}{e_2B-\underline{u}_{IG}(e_1,e_2)}$ and $1 \geq e_1 = \frac{e_2k}{e_2(B-\overline{u}_{IG}(e_1,e_2))-k}$. Because, whenever

$B > 2k + u_{IG}$, $\frac{2k}{e_2 B - u_{IG}} > \frac{e_2 k}{e_2(B-u_{IG})-k}$ for all $e_2 \in \left[\frac{2k+u_{IG}}{B}, 1\right]$, we have $\overline{u}_{IG}(e_1, e_2) > \underline{u}_{IG}(e_1, e_2)$. Moreover, as $\frac{2k}{e_2 B - u_{IG}}$ is increasing on $[0, e_2 B]$, we have $e_1 < \frac{2k}{e_2 B - u_{IG}}$ for all $u_{IG} > \underline{u}_{IG}(e_1, e_2)$, which implies that the Agent does not exert effort on both tasks under bundling when $u_{IG} > \underline{u}_{IG}(e_1, e_2)$. Similarly, as $\frac{e_2 k}{e_2(B-u_{IG})-k}$ is increasing in u_{IG}, we have $e_1 \geq \frac{e_2 k}{e_2(B-u_{IG})-k}$ for all $u_{IG} \leq \overline{u}_{IG}(e_1, e_2)$ and effort is exerted on both tasks under unbundling. Thus, for any $x \in \mathcal{S}$, there exists u_{IG} such that unbundling has a strict incentive advantage over bundling.

2. Assume $x \in \mathcal{M} \cap \mathcal{U}$. Then, by Proposition A. 2 if the Agent is bribed under bundling he exerts effort on task 2. Hence, for the expected effort to be (weakly) higher on task 2 under unbundling, we need $e_2 B_2 - k \geq 0$, or equivalently $B_2 \geq k/e_2$. By Proposition A. 3 the bribe that IG needs to pay to A_1 to contract failure is $b_u = e_1 B_1 - k$. The allocation that maximizes b_u, while sustaining $a_2 = 1$ under unbundling is thus $(B_1 = B - k/e_2, B_2 = k/e_2)$. We then have $b_u = e_1(B - k/e_2) - k$. If the Principal uses the moderate retention rule, we have $b = e_1(1-e_2)B - k$ (see Lemma A. 4). Simple algebra establishes that $b \geq b_u$ if, and only if, $e_2 \leq \frac{\sqrt{k}}{\sqrt{B}}$. Suppose $e_2 < \frac{\sqrt{k}}{\sqrt{B}}$. Then, if $B_2 \geq k/e_2$ IG bribes the Agent either (1) under neither institution, or (2) under both, or (3) under unbundling but not under bundling. If $B_2 < k/e_2$, Agent A_2 does not exert effort on task 2 under unbundling. Hence, if $x \in \mathcal{M} \cap \mathcal{U}$, $e_2 < \frac{\sqrt{k}}{\sqrt{B}}$, the presence of IG can create a strict incentive advantage of bundling.

□

6 Interactions between Tasks (Proof of Proposition 6)

Proposition 6. *1. If $\gamma \in (\frac{1}{2}, \frac{B}{8k} + \frac{1}{2})$, then there exist vectors of parameters $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ for which bundling has a strict incentive advantage over unbundling and the Principal uses the moderate retention rule.*

2. *If $\gamma \in (\frac{B}{8k}+\frac{1}{2}, \frac{B}{2k})$ then there exist vectors of parameters $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ for which unbundling has a strict incentive advantage over bundling, but no vectors $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ for which bundling has a strict incentive advantage.*

Proof of Proposition 6. 1. Let e_2^U solve $e_2 = \frac{e_2 \gamma k}{e_2 B - \gamma k}$. We have $e_2^U = \frac{2\gamma k}{B}$. For all $e_2 < e_2^U$, we have $e_2 < \frac{e_2 \gamma k}{e_2 B - \gamma k}$. Therefore, for all (e_1, e_2) such that $e_1 = e_2$ and $e_2 < e_2^U$, $(a_1 = 1, a_2 = 1)$ cannot be sustained in equilibrium under unbundling. Now let e_2^L and e_2^H be the solutions to $1 - \frac{k(2\gamma-1)}{e_2 B} = \frac{k(2\gamma-1)}{(1-e_2)B - u_{IG}}$. We have

$$e_2^L = \frac{1}{2} - \frac{\sqrt{B}\sqrt{B - 4k(2\gamma - 1)}}{2B}$$

and

$$e_2^H = \frac{1}{2} + \frac{\sqrt{B}\sqrt{B - 4k(2\gamma - 1)}}{2B}.$$

Notice that if $\gamma < \frac{B}{8k} + \frac{1}{2}$, e_2^L and e_2^H are real numbers with $e_2^L < e_2^H$. Simple algebra also establishes that e_2^L and e_2^H also solve $e_2 = 1 - \frac{k(2\gamma-1)}{e_2 B}$ and, consequently, $e_2 = \frac{k(2\gamma-1)}{(1-e_2)B}$ and that $1 - \frac{k(2\gamma-1)}{e_2 B} > e_2 > \frac{k(2\gamma-1)}{(1-e_2)B}$ for all $e_2 \in (e_2^L, e_2^H)$. Hence, for all (e_1, e_2) such that $e_1 = e_2$ and $e_2 \in \left[e_2^L, e_2^H\right]$, there exists, following arguments given in the proof of Proposition 3, infinitely many vectors $(e_1^H, e_1^L, e_2^H, e_2^L, \pi)$ for which $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under bundling with the Principal using the moderate retention rule.

Simple algebra shows that if $\gamma = 1$ and $B > 4k$, then $e_2^L < e_2^U$. It follows that if $\gamma = 1$ and $B > 4k$, there exists infinitely many vectors (e_1, e_2) such that $e_1 = e_2$ and $e_2 \in \left[e_2^L, e_2^U\right)$, and for which, therefore, $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under bundling, but not under unbundling. Now note that e_2^L and e_2^H are well defined, continuous functions of γ if, and only if, $B - 4k(2\gamma - 1) > 0$, i.e. if, and only if, $\gamma < \frac{B}{8k} + \frac{1}{2}$. Moreover, if $B > 4k$, we have $\frac{B}{8k} + \frac{1}{2} > 1$. e_2^U is also a continuous function of γ. By continuity, there exists $\overline{\gamma} > 1$, such that for all $\gamma < \overline{\gamma}$, $e_2^L < e_2^H$ and $e_2^L < e_2^U$, which establishes the result.

2. Remember that for an equilibrium to exist in which the Agent exerts effort on both tasks under bundling, it must either (1) be the case that $1 - \frac{k(2\gamma-1)}{e_2 B} \geq e_1 \geq \frac{k(2\gamma-1)}{(1-e_2)B}$ or (2) the case that $e_1 \geq \frac{2\gamma k}{e_2 B}$. As shown in the proof of Proposition 6, if $\gamma > \frac{B}{8k} + \frac{1}{2}$ the first case cannot occur. In turn, $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under unbundling if, and only if, $1 \geq e_1 \geq \frac{e_2 \gamma k}{e_2 B - \gamma k}$. Notice that $e_1 \geq \frac{2\gamma k}{e_2 B}$ and $1 \geq e_1 \geq \frac{e_2 \gamma k}{e_2 B - \gamma k}$ can be satisfied if, and only if, $\gamma < \frac{B}{2k}$. Proceeding as in the proof of Proposition 3, we can show that for all values of (e_1, e_2) such that $1 \geq e_1 \geq \frac{2\gamma k}{e_2 B}$, we have $\frac{e_2 \gamma k}{e_2 B - \gamma k} < \frac{2\gamma k}{e_2 B}$ and therefore $1 \geq e_1 > \frac{e_2 \gamma k}{e_2 B - \gamma k}$. Finally, notice that if $B > 2k$, we have $\frac{B}{8k} + \frac{1}{2} < \frac{B}{2k}$.

□

7 Robustness: Continuous effort

In the model studied in the paper, we assume that each Agent can choose to exert effort $(a_i = 1)$ or not $(a_i = 0)$ and that exerting effort produces success with exogenous probability $e_i \in (0, 1)$. In this subsection, we consider the robustness of the arguments in favor of bundling in the context of a version of the model in which the effort choice a_i is continuous and the probability of success e_i is an increasing function of effort. The model is identical to the main model in the paper up to the following modifications: (1) we now assume that $0 < \theta_L < \theta_H$, (2) $a_i \geq 0$ and the cost of effort is a_i^2, and (3) the probability of success is $e_i := c_i(\pi\theta_H + (1-\pi)\theta_L)a_i$, with $c_i > 0$. We interpret the new parameters $c_1, c_2 > 0$ as measures of complexity.

As the purpose of this analysis is to consider the robustness of the arguments with respect to bundling, we suppress the full characterization, and focus on the following result:

Proposition A. 4. *Regardless of action transparency, for any parameter values* $B, \pi, \theta_L, \theta_H$, *there exists a level of complexity* $\tilde{c} \leq \frac{\sqrt{2}}{\sqrt{BT}}$ *such that for all* $c_1, c_2 < \tilde{c}$,

1. *in the equilibrium that maximizes the total effort of the Agent under bundling, the Agent exerts positive effort on both tasks and the Principal uses the moderate retention rule;*

2. *under this moderate incentives equilibrium, total effort is higher under bundling than under unbundling for any pair* (B_1, B_2) *of values of holding office.*

Proof of Proposition A. 4. 1. We proceed in a number of steps. We first show that if c_1 and c_2 are sufficiently low, the moderate incentives equilibrium is the only equilibrium to sustain positive effort on both tasks. We then show that if c_1 and c_2 are sufficiently low, total effort is higher in the moderate incentives equilibrium than in an equilibrium in which the Agent exerts positive effort only on task i.

We denote $(\mathring{a}_1, \mathring{a}_2)$ the Principal's expectations about the Agent's actions when the Principal does not observe the actions (a_1, a_2) chosen by the Agent. The Principal's posterior belief about the Agent's competence, upon observing outcomes (o_1, o_2) and given expectations $(\mathring{a}_1, \mathring{a}_2)$, is then denoted $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$. Similarly, the Principal's posterior, upon observing outcomes (o_1, o_2) and effort choices (a_1, a_2), is denoted $Pr(\theta = \theta_H | (o_1, o_2); (a_1, a_2))$. Note that we have $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_1, o_2); (a_1, a_2))$ whenever $(o_1, o_2); (\mathring{a}_1, \mathring{a}_2) = (o_1, o_2); (a_1, a_2)$. To simplify notation, we thus only look at $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$ in the sequel of this proof. Remember that it is a best response for the Principal to retain the Agent if, and only if, $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) \geq \pi$.

Suppose first that the Agent exerts effort levels $(\mathring{a}_1, \mathring{a}_2)$, with $\mathring{a}_1, \mathring{a}_2 > 0$. We then have

$$Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (\mathring{a}_1, \mathring{a}_2)) = \frac{\theta_H^2 c_1 \mathring{a}_1 c_2 \mathring{a}_2 \pi}{\theta_H^2 c_1 \mathring{a}_1 c_2 \mathring{a}_2 \pi + \theta_L^2 c_1 \mathring{a}_1 c_2 \mathring{a}_2 (1 - \pi)},$$

$$Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (\mathring{a}_1, \mathring{a}_2)) = \frac{(1-\theta_H c_1 \mathring{a}_1)(1-\theta_H c_2 \mathring{a}_2)\pi}{(1-\theta_H c_1 \mathring{a}_1)(1-\theta_H c_2 \mathring{a}_2)\pi + (1-\theta_L c_1 \mathring{a}_1)(1-\theta_L c_2 \mathring{a}_2)(1-\pi)},$$

and

$$Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_1, \mathring{a}_2)) = \frac{\theta_H c_i \mathring{a}_i (1-\theta_H c_j \mathring{a}_j)\pi}{\theta_H c_i \mathring{a}_i (1-\theta_H c_j \mathring{a}_j)\pi + \theta_L c_i \mathring{a}_i (1-\theta_L c_j \mathring{a}_j)(1-\pi)}.$$

Because $\theta_H > \theta_L > 0$ we have $Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (a_1, a_2)) > \pi$ and $Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (a_1, a_2)) < \pi$. Consequently, the Principal's best response is to retain upon observing $(o_1 = s, o_2 = s)$, and to dismiss upon observing $(o_1 = f, o_2 = f)$. In turn, $Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_i = s, o_j = f); (a_1, a_2)) \geq \pi$ if, and only if, $\mathring{a}_j < \frac{1}{c_j(\theta_H + \theta_L)}$ (respectively $a_j < \frac{1}{c_j(\theta_H + \theta_L)}$). Consequently, the Principal's best response is to retain upon observing $(o_i = s, o_j = f)$ for all $i = 1, 2$, $j \neq i$, if, and only if, $\mathring{a}_j < \frac{1}{c_j(\theta_H + \theta_L)}$ (respectively $a_j < \frac{1}{c_j(\theta_H + \theta_L)}$) for all $j = 1, 2$.

Define $T := (\pi\theta_H + (1-\pi)\theta_L)$ and suppose the Principal uses the moderate retention rule. The Agent then chooses a_1, a_2, so as to maximize $(c_1 T a_1 + c_2 T a_2 - T^2 c_1 a_1 c_2 a_2)B - a_1^2 - a_2^2$ subject to the constraints that $a_1, a_2 \geq 0$.

Using the Kuhn-Tucker approach, we set up the Lagrangean

$$L(a_1, a_2, \lambda_1, \lambda_2) = (c_1 T a_1 + c_2 T a_2 - T^2 c_1 a_1 c_2 a_2)B - a_1^2 - a_2^2 + \lambda_1 a_1 + \lambda_2 a_2.$$

Note that if, $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{B}T}$, the objective function, and the inequality constraints are concave in (a_1, a_2). Thus, the first order conditions are necessary and sufficient for a maximum. The first order conditions are:

$$\frac{\partial L}{\partial a_1} = c_1 T(1 - T c_2 a_2)B - 2a_1 + \lambda_1 = 0 \tag{1}$$

$$\frac{\partial L}{\partial a_2} = c_2T(1 - Tc_1a_1)B - 2a_2 + \lambda_2 = 0 \tag{2}$$

$$\lambda_1 a_1 = 0, \quad \lambda_2 a_2 = 0, \quad a_1, a_2 \geq 0, \quad \lambda_1, \lambda_2 \geq 0.$$

(a) Case 1: WLOG suppose $\lambda_1 > 0$, which implies, by $\lambda_1 a_1 = 0$, that $a_1 = 0$. Solving equation (1) for λ_1, given that $a_1 = 0$, we find $\lambda_1 = -c_1T(1 - Tc_2a_2)B$ which is greater or equal to 0 if, and only if, $a_2 \geq \frac{1}{Tc_2}$.

In turn, solving equation (2) for a_2, given $a_1 = 0$, we get $a_2 = \frac{BTc_2+\lambda_2}{2}$. Because, $\lambda_2 \geq 0$ and $BTc_2 > 0$, we have $a_2 > 0$. $\lambda_2 a_2 = 0$, in turn, implies $\lambda_2 = 0$ and thus $a_2 = \frac{BTc_2}{2}$. If $c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, we have $a_2 = \frac{Bc_2T}{2} < \frac{1}{c_2T}$. Consequently, for $c_2 < \tilde{c}$, there is no solution in which $\lambda_1 > 0$. A similar argument shows that there is no solution with $\lambda_2 > 0$.

(b) Case 2: Hence, we now consider the case $\lambda_1 = \lambda_2 = 0$. Solving equations (1) and (2) for a_1 and a_2, we get

$$a_1 = \frac{Bc_1T(2 - Bc_2^2T^2)}{4 - B^2c_1^2c_2^2T^4}, \quad a_2 = \frac{Bc_2T(2 - Bc_1^2T^2)}{4 - B^2c_1^2c_2^2T^4}.$$

Notice that if $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, we have $2 - Bc_2^2T^2 < 0$, $2 - Bc_1^2T^2 < 0$, and $4 - B^2c_1^2c_2^2T^4 < 0$. Consequently, for $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, we have $a_1, a_2 > 0$. We conclude that, $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$,

$$a_1 = \frac{Bc_1T(2 - Bc_2^2T^2)}{4 - B^2c_1^2c_2^2T^4}, \quad a_2 = \frac{Bc_2T(2 - Bc_1^2T^2)}{4 - B^2c_1^2c_2^2T^4},$$

is the Agent's best response to the moderate retention rule.

Remember that the moderate retention rule, in turn, is a best response to the Agent's effort allocation if, and only if, $a_i < \frac{1}{c_i(\theta_H+\theta_L)}$ for all $i = 1, 2$. Note that, in any equilibrium in which the Agent exerts positive effort on both tasks, a_1

and a_2 are strictly increasing in c_1, and c_2, respectively. Hence, if c_1, c_2, are sufficiently low, we have $a_i < \frac{1}{c_i(\theta_H+\theta_L)}$ for all $i = 1, 2$. Consequently, there exists $\tilde{c} \leq \frac{\sqrt{2}}{\sqrt{BT}}$, such that if $c_1, c_2 \leq \tilde{c}$ the moderate incentives equilibrium is the unique equilibrium in which the Agent exerts effort on both tasks.

We now show that if $c_1, c_2 \leq \tilde{c}$, the moderate incentives equilibrium maximizes the Principal's welfare. To do so we now derive the effort level of the Agent in the equilibrium in which the Agent exerts effort solely on task i. Given such an effort allocation, the Principal retains the Agent if, and only if, $o_i = s$. The Agent thus maximizes $a_i c_i T B - a_i^2$ subject to the constraint that $a_i \geq 0$, which yields $a_i = \frac{BTc_i}{2}$ as a maximizer. Simple, but tedious algebra, shows that if $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, then $\frac{Bc_1T(2-Bc_2^2T^2)}{4-B^2c_1^2c_2^2T^4} + \frac{Bc_2T(2-Bc_1^2T^2)}{4-B^2c_1^2c_2^2T^4} > \frac{BTc_i}{2}$ for all $i = 1, 2$. Consequently, for $c_1, c_2 \leq \tilde{c}$ total effort is higher in the moderate incentives equilibrium than in any $i - th$ task equilibrium.

2. Under unbundling, the Principal retains Agent A_i if, and only if $o_i = s$. Agent A_i thus maximizes $a_i c_i T B_i - a_i^2$, which yields $a_i = \frac{B_i c_i T}{2}$. Total effort under unbundling is thus equal to $\frac{B_1 c_1 T}{2} + \frac{(B-B_1)c_2 T}{2}$. Maximizing this total effort with respect to B_1, yields $(a_1 = 0, a_2 = \frac{Bc_2T}{2})$ if $c_1 \leq c_2$ and $(a_1 = \frac{Bc_1T}{2}, a_2 = 0)$ if $c_1 > c_2$. Notice that maximal total effort under unbundling is thus equal to effort in the $i-th$ task equilibrium under bundling. It follows that, for $c_1, c_2 \leq \tilde{c}$ total effort is higher in the moderate incentives equilibrium under bundling than in the optimal (B_1, B_2) allocation equilibrium under unbundling.

□