Does Unbundling Policy Authority Improve Accountability?

Appendix for online publication

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1 Bundling (Proof of Proposition 1)

Proposition 1. *On the equilibrium path of play under bundling:*

1. *the Agent chooses to exert effort on both tasks if, and only if, the pair of policy-making environment and beliefs satisfies either of the following two sets of conditions:*

   (a) *(i) the complexity of each task is sufficiently low,* $p_i \geq \frac{2k}{p_j B}$*;*

   (ii) the Principal expects the Agent to exert effort in both policy areas; and

   (iii) the Principal's estimation of the Agent's competence decreases unless the outcome is success on both tasks, $p_i^H(1-p_j^H) \leq p_i^L(1-p_j^L)$ *for all* $i = 1, 2$*;*

   or

   (b) *(i) the complexity of each task is moderate,* $1 - \frac{k}{p_j B} \geq p_i \geq \frac{k}{(1-p_j)B}$*; and*

   (ii) the Principal's estimation of the Agent's competence increases when the outcome is success on at least one task, $p_i^H(1-p_j^H) \geq p_i^L(1-p_j^L)$ *for all* $i = 1, 2$*.*

   When conditions (1) hold, the Principal adopts the strict retention rule, and when conditions (2) hold, the moderate retention rule.

2. *The Agent chooses to exert effort only on task* i *if, and only if, all of the following three conditions hold*

   (a) *the complexity of that task is sufficiently low,* $p_i \geq k/B$*,*

   (b) *it is (weakly) lower than the complexity of task* $j \neq i$*,* $p_i \geq p_j$*, and*

   (c) *the conditions to sustain an equilibrium in which the Agent chooses to exert effort on both tasks and the Principal uses a moderate retention rule (stated in part 1.(b)) are not satisfied.*

   In this equilibrium where the Agent exerts effort only on task i*, the Principal uses the moderate retention rule.*

3. *The Agent chooses to exert no effort on either task when the complexity of each task is sufficiently high,* $p_i < k/B$ *for all* $i = 1, 2$*, independent of the Principal's retention rule.*

To prove Proposition 1, we first derive, in Lemma A. 1 the best response of the Principal to effort allocations chosen by the Agent. We then derive, in Lemma A. 2, the Agent's best response effort allocation to retention rules used by the Principal.

Lemma A. 1 (Best-response of the Principal)**.** 1. *If the Principal expects that the Agent chooses* $(a_1 = 1, a_2 = 1)$ *or if the Principal observes that the Agent chooses* $(a_1 = 1, a_2 = 1)$*, then the Principal retains the Agent upon observing* $(o_1 = s, o_2 = s)$*, retains upon observing* $(o_i = s, o_j = f)$ *if, and only if,* $p_i^H(1-p_j^H) \geq p_i^L(1-p_j^L)$ *for all* $i = 1, 2$*, and does not retain upon observing* $(o_1 = f, o_2 = f)$*.*

2. *If the Principal expects that the Agent chooses* $(a_i = 1, a_j = 0)$ *or if the Principal observes that the Agent chooses* $(a_i = 1, a_j = 0)$*, then it is a best-response for the Principal to reelect if* $(o_i = s, o_j = f)$ *and to dismiss if* $(o_i = f, o_j = f)$*.*

3. *If the Principal expects that the Agent chooses* $(a_1 = 0, a_2 = 0)$ *or if the Principal observes that the Agent chooses* $(a_1 = 0, a_2 = 0)$*, then the Principal is indifferent between retaining and dismissing the Agent upon observing* $(o_1 = f, o_2 = f)$*. Outcomes* $(o_1 = s, o_2 = s)$*,* $(o_1 = f, o_2 = s)$*, and* $(o_1 = s, o_2 = f)$ *are off the equilibrium path in this case and any retention decision is a best response a these information sets.*

Before proving Lemma A. 1, we note that, unless the Agent exerts effort on both tasks and $p_i^H(1 - p_j^H) < p_i^L(1 - p_j^L)$ for some $i = 1, 2$, the moderate retention rule always is a best-response.

Proof of Lemma A. 1. We denote $(\mathring{a}_1, \mathring{a}_2)$ the Principal's expectations about the Agent's actions when the Principal does not observe the actions (a_1, a_2) chosen by the Agent.

The Principal's posterior belief about the Agent's competence, upon observing outcomes (o_1, o_2) and given expectations $(\mathring{a}_1, \mathring{a}_2)$, is then denoted $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$. Similarly, the Principal's posterior, upon observing outcomes (o_1, o_2) and effort choices (a_1, a_2), is denoted $Pr(\theta = \theta_H|(o_1, o_2); (a_1, a_2))$. Note that we have $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H|(o_1, o_2); (a_1, a_2))$ whenever $(o_1, o_2); (\mathring{a}_1, \mathring{a}_2) = (o_1, o_2); (a_1, a_2)$. To simplify notation, we thus only look at $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$ in the sequel of this proof. Remember that it is a best response for the Principal to retain the Agent if, and only if, $Pr(\theta = \theta_H|(o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) \geq \pi$. We have

$$Pr(\theta = \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = \frac{p_1^H p_2^H \pi}{p_1^H p_2^H \pi + p_1^L p_2^L (1 - \pi)},$$

$$Pr(\theta = \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = \frac{(1 - p_1^H)(1 - p_2^H)\pi}{(1 - p_1^H)(1 - p_2^H)\pi + (1 - p_1^L)(1 - p_2^L)(1 - \pi)},$$

and

$$Pr(\theta = \theta_H|(o_i = s, o_j = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = \frac{p_i^H (1 - p_j^H)\pi}{p_i^H (1 - p_j^H)\pi + p_i^L (1 - p_j^L)(1 - \pi)}.$$

Because $p_i^H > p_i^L$ for all $i = 1, 2$, we have $Pr(\theta = \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = Pr(\theta = \theta_H|(o_1 = s, o_2 = s); (a_1 = 1, a_2 = 1)) > \pi$ and $Pr(\theta = \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = Pr(\theta = \theta_H|(o_1 = f, o_2 = f); (a_1 = 1, a_2 = 1)) < \pi$. Consequently, the Principal's best response is to retain upon observing $(o_1 = s, o_2 = s)$, and to dismiss upon observing $(o_1 = f, o_2 = f)$. In turn, $Pr(\theta = \theta_H|(o_i = s, o_j = f); (\mathring{a}_1 = 1, \mathring{a}_2 = 1)) = Pr(\theta = \theta_H|(o_i = s, o_j = f); (a_1 = 1, a_2 = 1)) \geq \pi$ if, and only if, $p_i^H(1 - p_j^H) \geq p_i^L(1 - p_j^L)$. Consequently, the Principal's best response is to retain upon observing $(o_i = s, o_j = f)$ if, and only if, $p_i^H(1 - p_j^H) \geq p_i^L(1 - p_j^L)$. Similarly, we have

$$Pr(\theta = \theta_H|(o_i = s, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = \frac{p_i^H \pi}{p_i^H \pi + p_i^L (1 - \pi)},$$

and

$$Pr(\theta = \theta_H | (o_i = f, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = \frac{(1 - p_i^H)\pi}{(1 - p_i^H)\pi + (1 - p_i^L)(1 - \pi)}.$$

Because $p_i^H > p_i^L$ for all $i = 1, 2$, we have $Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = Pr(\theta = \theta_H | (o_i = s, o_j = f); (a_i = 1, a_j = 0)) > \pi$ and $Pr(\theta = \theta_H | (o_i = f, o_j = f); (\mathring{a}_i = 1, \mathring{a}_j = 0)) = Pr(\theta = \theta_H | (o_i = f, o_j = f); (a_i = 1, a_j = 0)) < \pi$. Consequently the Principal's best response is to retain upon observing $(o_i = s, o_j = f)$, and to dismiss upon observing $(o_i = f, o_j = f)$.

Finally,

$$Pr(\theta = \theta_H | (o_i = f, o_j = f); (\mathring{a}_i = 0, \mathring{a}_j = 0)) = \pi,$$

which implies that the Voter is indifferent between retaining and dismissing the Incumbent. Outcomes $(o_1 = s, o_2 = s)$, $(o_1 = f, o_2 = s)$, and $(o_1 = s, o_2 = f)$ are off the equilibrium path in this case and any retention decision is a best response a these information sets. □

Lemma A. 2 (Best-response of the Agent)**.**

1. *If the Principal retains the Agent if, and only if,* $(o_1 = s, o_2 = s)$*, then the Agent's best response is to choose* $(a_1 = 1, a_2 = 1)$ *if* $p_1 p_2 B - 2k \geq 0$ *and to choose* $(a_1 = 0, a_2 = 0)$ *if* $p_1 p_2 B - 2k \leq 0$.

2. *If the Principal retains the Agent if, and only if,* $o_i = s$ *for at least some* $i = 1, 2$*, then the Agent's best response is (1) to choose* $(a_1 = 1, a_2 = 1)$ *if* $p_i(1 - p_j)B - k \geq 0$ *for all* $i = 1, 2$, $j \neq i$*, (2) to choose* $(a_i = 1, a_j = 0)$ *if* $[0 \geq p_j(1 - p_i)B - k, p_i \geq p_j,$ *and* $p_i B - k \geq 0]$*, and (3) to choose* $(a_1 = 0, a_2 = 0)$ *if* $p_i B - k \leq 0$ *for all* $i = 1, 2$.

3. *If the Principal retains the Agent if, and only if,* $o_i = s$ *for specific* $i = 1, 2$*, then the Agent's best response is to choose* $(a_i = 1, a_j = 0)$ *if* $p_i B - k \geq 0$*, and to choose* $(a_i = 0, a_j = 0)$ *if* $p_i B - k \leq 0$.

4. *If the Principal always or never retains the Agent, then the Agent's best response is to choose* $(a_1 = 0, a_2 = 0)$.

Proof of Lemma A. 2. Suppose first that the Principal retains the Agent if, and only if, $(o_1 = s, o_2 = s)$. Let us denote this retention rule by r_s. Then

$$U_A((a_1 = 1, a_2 = 1), r_s) = p_1 p_2 B - 2k$$

$$U_A((a_i = 1, a_j = 0), r_s) = -k \quad \text{for all } i = 1, 2, j \neq i$$

$$U_A((a_1 = 0, a_2 = 0), r_s) = 0.$$

Choosing $(a_i = 1, a_j = 0)$ is never a best response because $-k < 0$. Hence, choosing $(a_1 = 1, a_2 = 1)$ is the Agent's best response to the strict retention rule if, and only if, $p_1 p_2 B - 2k \geq 0$, while choosing $(a_1 = 0, a_2 = 0)$ is the Agent's best response if, and only if, $p_1 p_2 B - 2k \leq 0$.

Suppose now that the Principal retains the Agent if, and only if, $o_i = s$ for at least some $i = 1, 2$. Let us denote this retention rule by r_m. Then

$$U_A((a_1 = 1, a_2 = 1), r_m) = (p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2)B - 2k$$

$$U_A((a_1 = 1, a_j = 0), r_m) = p_1 B - k$$

$$U_A((a_1 = 0, a_2 = 1), r_m) = p_2 B - k$$

$$U_A((a_1 = 0, a_2 = 0), r_m) = 0.$$

Hence, choosing $(a_1 = 1, a_2 = 1)$ is the Agent's best response to the moderate retention rule if, and only if, the following two conditions are satisfied: (1) $(p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2)B - 2k \geq p_i B - k$ for all $i = 1, 2$, and (2) $(p_1 p_2 + p_1(1 - p_2) + (1 - p_1)p_2)B - 2k \geq 0$. Condition (1), in turn, is satisfied if, and only if, $[p_1(1 - p_2)B - k \geq 0,$ and $(1 - p_1)p_2 B - k \geq 0]$ which implies condition (2). Choosing $(a_i = 1, a_j = 0)$ is the Agent's best response if the following three conditions hold: (1) $p_i B - k \geq 0$, (2) $p_i B - k \geq p_j B - k$, and (3) $p_i B - k \geq (p_1 p_2 + p_1(1 -$

$p_2)+(1-p_1)p_2)B-2k$ which is equivalent to $[0 \geq p_j(1-p_i)B-k, p_i \geq p_j,$ and $p_iB-k \geq 0]$. Finally, choosing $(a_i = 0, a_j = 0)$ is the Agent's best response if the following two conditions hold: (1) $0 \geq p_iB - k$ for all $i = 1, 2$, and (2) $0 \geq (p_1p_2 + p_1(1-p_2) + (1-p_1)p_2)B - 2k$. Because (1) implies (2), this is equivalent to $p_iB - k \leq 0$ for all $i = 1, 2$.

Suppose next that the Principal retains the Agent if, and only if, $o_i = s$ for a specific $i = 1, 2$. We call this retention rule the i^{th}-task retention rule and denote it by r_i. Then

$$U_A((a_1 = 1, a_2 = 1), r_i) = p_iB - 2k$$
$$U_A((a_i = 1, a_j = 0), r_i) = p_iB - k$$
$$U_A((a_i = 0, a_j = 1), r_i) = -k$$
$$U_A((a_1 = 0, a_2 = 0), r_i) = 0.$$

Hence, choosing $(a_1 = 1, a_2 = 1)$ is never a best-response to the i^{th}-task retention rule because $p_iB - 2k < p_iB - k$. Similarly, $(a_i = 0, a_j = 1)$ is never a best response because $-k < 0$. Choosing $(a_i = 1, a_j = 0)$ is thus the Agent's best response to the i^{th}-task retention rule if, and only if, $p_iB - k \geq 0$, while choosing $(a_i = 0, a_j = 0)$ is the Agent's best response if, and only if, $p_iB - k \leq 0$.

If the Principal never retains (or always retains), then $U_A(a_1 = 1, a_2 = 1) < U_A(a_i = 1, a_j = 0) < U_A(a_1 = 0, a_2 = 0)$ and the Agent's best response is to choose $(a_1 = 0, a_2 = 0)$. □

Remark. Comparing the best-response of the Agent to the moderate retention rule (case 2. in Lemma A. 2) and the best-response of the Agent to the retention rule where the Principal retains if, and only, $o_i = s$ (i^{th}-task retention rule, case 3. in Lemma A. 2) shows that this last retention rule is weakly dominated by the moderate retention rule, whenever (1) the moderate retention sustains effort into both tasks by the Agent (case 2.(1) in Lemma A. 2) or (2) the moderate retention rule incentivizes effort into the task i with higher probability of success ($p_i \geq p_j$) and task i also provides the Principal with a more precise signal of the Agent's competence. When that is not the case, the moderate retention rule may not weakly dominate the i^{th}-task retention rule (only) if the latter requires the Agent to invest effort into a task with lower probability of success. However, when that is the case, the i^{th}-task rule will not be robust. To see this, note that if the Agent were to deviate by exerting effort on the less complex task, the only inference available to the Principal is that the Agent chose to exert effort on that task. Once the Principal updates accordingly, it is no longer rational for her to follow the i^{th}-task rule. With the conjunction of weak dominance and this argument, we restrict attention to the equilibria with moderate or strict rules.

Proof of Proposition 1. Follows from Lemmata A. 1 and A. 2 by looking for mutual best responses, taking into account the remark above. □

The following Lemma shows that the conditions on (p_1, p_2) and on $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ stated in Proposition 1, that are necessary to sustain an equilibrium in which the Agent exerts effort on both tasks, can jointly be satisfied.

Lemma A. 3. *1. For any $(p_1, p_2) \in (0,1)^2$ there exists an infinity of $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ such that $p_i = \pi p_i^H + (1-\pi)p_i^L$ and $p_i^H(1-p_j^H) \geq p_i^L(1-p_j^L)$ for all $i, j = 1, 2, j \neq i$.*

2. For any $(p_1, p_2) \in (0,1)^2$ there exists an infinity of $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ such that $p_i = \pi p_i^H + (1-\pi)p_i^L$ and $p_i^H(1-p_j^H) \leq p_i^L(1-p_j^L)$ for all $i, j = 1, 2, j \neq i$.

Proof of Lemma A. 3. 1. Suppose first $p_1 = p_2 \in (0,1)$. Note that for any $p_1^H, p_1^L, p_2^H, p_2^L$ such that (1) $p_1^H = p_2^H$, (2) $p_1^L = p_2^L$, (3) $p_1^L < p_1 < p_1^H$, and (4) $|p_1^H - 1/2| \leq |p_1^L - 1/2|$, we have $p_i^H(1-p_j^H) \geq p_i^L(1-p_j^L)$ for all $i, j = 1, 2, j \neq i$. It is easy to see that there is an infinity of $p_1^H, p_1^L, p_2^H, p_2^L$ that satisfy conditions (1) through (4). Now let $\pi = \frac{p_i - p_i^L}{p_i^H - p_i^L}$. Because $p_i^H > p_i > p_i^L$, we have $\pi \in (0,1)$. Moreover, some simple algebra establishes that $p_i = \pi p_i^H + (1-\pi)p_i^L$. WLOG suppose next that $1 > p_1 > p_2 > 0$. Some simple algebra establishes that $p_i^H(1-p_j^H) > p_i^L(1-p_j^L)$ for all $i, j = 1, 2, j \neq i$ if, and only if $p_1^H < 1 - p_2^L$ and $p_1^L < 1 - p_2^H$. Choose any p_1^H and p_2^L such that (1) $p_1^H < 1 - p_2^L$, (2) $p_2 > p_2^L > 0$, and (3) $\frac{1-p_2^L}{p_2-p_2^L}p_1 > p_1^H > p_1$. It is easy to see that there is an infinity of p_1^H's and p_2^L's that satisfy conditions (1) through (3). Now choose $\pi \in (\frac{p_2-p_2^L}{1-p_2^L}, \frac{p_1}{p_1^H}) \subset (0,1)$ close enough to $\frac{p_1}{p_1^H}$. By condition (3) we have $\frac{p_2-p_2^L}{1-p_2^L} < \frac{p_1}{p_1^H}$. Therefore such a value of π exists. Now let $p_1^L = \frac{p_1 - \pi p_1^H}{1-\pi}$ and $p_2^H = \frac{p_2-(1-\pi)p_2^L}{\pi}$. By this definition of p_1^L and p_2^H, we have $p_i = \pi p_i^H + (1-\pi)p_i^L$ for all $i = 1, 2$. Moreover, because $\pi < \frac{p_1}{p_1^H}$, we have $p_1^L > 0$. Similarly, $\pi > \frac{p_2-p_2^L}{1-p_2^L}$ implies that $1 > p_2^H$. Finally, we have $p_2^H < 1 - p_1^L$ if, and only if, $\frac{p_2-(1-\pi)p_2^L}{\pi} < 1 - \frac{p_1-\pi p_1^H}{1-\pi}$, if, and only if, $f(\pi) := \pi^2(1-p_2^L-p_1^H) + \pi(-1-p_2+2p_1^L+p_1) + p_2 - p_2^L < 0$. As $p_1^H < 1$, which by assumption is lower than $\frac{p_1}{p_2}$, some simple algebra shows that $f(\frac{p_1}{p_1^H}) < 0$. As $f(\pi)$ is a continuous function of π, we have $f(\pi) < 0$ for any value of π close enough to $\frac{p_1}{p_1^H}$, which establishes the result.

2. Proceeding in a similar way than just above yields the result

□

2 Comparing Institutions (Proof of Proposition 2)

Proposition 2 (***Strict Incentive Advantage***). *1. The set of policy-making environments for which bundling has a strict incentive advantage over unbundling has policy-area complexities which are relatively high on both dimensions.*

2. If bundling has a strict incentive advantage over unbundling, effort in both policy areas under bundling is sustainable in equilibrium only under the moderate retention rule.

3. The set of policy-making environments for which unbundling has a strict incentive *advantage over bundling has asymmetric policy-area complexities: sufficiently high for one area and intermediate for the other.*

Proof of Proposition 2. 1. We first establish that there is a set of policy-making environments $\boldsymbol{s} := (p_1^H, p_1^L, p_2^H, p_2^L, \pi, k, B) \in \boldsymbol{\Sigma}$, for which $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under bundling, but not under unbundling. Remember that there exists (B_1, B_2) such that both Agents exert effort under unbundling if, and only if, $1 \geq p_1 \geq \frac{p_2 k}{p_2 B - k}$. Let p^u be the intersection between $p_1 = \frac{p_2 k}{p_2 B - k}$ and $p_1 = p_2$. We have $p^u = \frac{2k}{B}$. Note that $p^u < 1$ if $B > 4k$. For the rest of the paragraph we restrict attention to values of (p_1, p_2) such that $p_1 = p_2$. Abusing notation slightly, we work with $p = p_1 = p_2$. For any $p < p^u$, effort is exerted on at most one task under unbundling because we have $p < \frac{pk}{pB-k}$ whenever $p < p^u$. In the case of symmetric probabilities of success $p_1 = p_2$, condition 1.(b) in Proposition 1 and Lemma A. 3 imply that if $p(1-p)B - k \geq 0$ effort on both tasks can be sustained in equilibrium under bundling. Hence, we now derive the set of values of p that satisfy $p(1-p)B - k \geq 0$. We have $p(1-p)B - k = 0$ for $\frac{1}{2} - \frac{\sqrt{B(B-4k)}}{2B} := \underline{p}^m$ and for $\frac{1}{2} + \frac{\sqrt{B(B-4k)}}{2B} := \overline{p}^m$. If $B > 4k$, $\underline{p}^m$ and $\overline{p}^m$ are well-defined and $0 \leq \underline{p}^m < 1/2 < \overline{p}^m \leq 1$. Moreover, $p(1-p)B - k$ reaches its maximum at $p = 1/2$ and is strictly increasing on $[0, 1/2)$ and strictly decreasing on $(1/2, 1]$. Thus, for all $p \in \left[\underline{p}^m, \overline{p}^m\right]$, we have $p(1-p)B - k \geq 0$. Simple algebra shows that $\underline{p}^m < p^u$ if $B > 4k$. It follows that for all intermediate levels of policy area complexities $p \in \left[\underline{p}^m, p^u\right)$, $(a_1 = 1, a_2 = 1)$ cannot be sustained under unbundling, but can be sustained under bundling, provided the moderate retention rule is sequentially rational. By Lemma A. 3 there exists for any $(p_1, p_2) \in (0, 1)^2$ an infinity of $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ such that $p_i = \pi p_i^H + (1-\pi) p_i^L$ and $p_i^H(1 - p_j^H) \geq p_i^L(1 - p_j^L)$ for all $i, j = 1, 2, j \neq i$.

2. We now establish that for all parameter values such that there exists an equilibrium

in which the Agent exerts high effort on both tasks under bundling with the Principal using the strict retention rule, there exists a feasible pair (B_1, B_2) such that both Agents exert effort under unbundling. Consequently, for any policy-making environment $\boldsymbol{s} := (p_1^H, p_1^L, p_2^H, p_2^L, \pi, k, B) \in \boldsymbol{\Sigma}$, such that, in equilibrium, effort is exerted on both tasks under bundling, yet not under unbundling, it must be the case that the Principal uses the moderate retention rule. If the Principal uses the strict retention rule, the Agent chooses $(a_1 = 1, a_2 = 1)$ if, and only if, $p_1 p_2 B - 2k \geq 0$ or equivalently $1 \geq p_1 \geq \frac{2k}{p_2 B}$. Similarly, there exists (B_1, B_2) such that both Agents exert effort under unbundling if, and only if, $1 \geq p_1 \geq \frac{p_2 k}{p_2 B - k}$. We now prove that for all $(p_1, p_2) \in [0,1]^2$, if $1 \geq p_1 \geq \frac{2k}{p_2 B}$ then $1 \geq p_1 \geq \frac{p_2 k}{p_2 B - k}$. Note first that if $0 \leq p_2 < \frac{2k}{B}$, then $\frac{2k}{p_2 B} > 1$ and there is no value of p_1 that satisfies $1 \geq p_1 \geq \frac{2k}{p_2 B}$. Similarly, if $0 < B < 2k$ there is no $(p_1, p_2) \in [0,1]^2$ that satisfies $1 \geq p_1 \geq \frac{2k}{p_2 B}$. We have $\frac{p_2 k}{p_2 B - k} < \frac{2k}{p_2 B}$ if, and only if, $Q(p_2) := p_2^2 B - 2p_2 B + 2k < 0$. If $p_2 = \frac{2k}{B}$, then $Q(p_2) \leq 0$ whenever $B \geq 2k$. Moreover, $\frac{dQ}{dp_2} = 2p_2 B - 2B < 0$ for all $p_2 \in [0, 1)$. Hence, $\frac{p_2 k}{p_2 B - k} \leq \frac{2k}{p_2 B}$ for all $p_2 \in \left[\frac{2k}{B}, 1\right]$, which establishes the claim.

3. We finally prove that unbundling can have a strict incentive advantage over bundling and that this occurs when the complexity of policy areas is sufficiently asymmetric. Remember that there exists (B_1, B_2) such that both Agents exert effort under unbundling if, and only if, $1 \geq p_1 \geq \frac{p_2 k}{p_2 B - k}$. Similarly, there does not exist an equilibrium in which the Agent exerts effort on both tasks if $\left[(1)\ p_1 < \frac{2k}{p_2 B} \text{ and } (2)\ p_1 \notin \left[\frac{k}{(1-p_2)B}, 1 - \frac{k}{p_2 B}\right]\right]$. Condition (1) implies that there is no equilibrium in which the Agent chooses $(a_1 = 1, a_2 = 1)$ and the Principal uses the strict retention rule, while (2) implies that there is no equilibrium in which the Agent chooses $(a_1 = 1, a_2 = 1)$ and the Principal uses the moderate retention rule. Hence, to show that unbundling can have a strict incentive advantage we need to prove that there exist $\boldsymbol{s} := (p_1^H, p_1^L, p_2^H, p_2^L, \pi, k, B) \in \boldsymbol{\Sigma}$, such

that $\frac{2k}{p_2 B} > p_1 \geq \frac{p_2 k}{p_2 B - k}$ and $p_1 \notin \left[\frac{k}{(1-p_2)B}, 1 - \frac{k}{p_2 B}\right]$. Let $p_2 = \frac{2k}{B}$. Then (i) $\frac{2k}{p_2 B} = 1$, (ii) $1 - \frac{k}{p_2 B} = 1/2$, (iii) $\frac{k}{(1-p_2)B} = \frac{k}{B-2k}$, and (iv) $\frac{p_2 k}{p_2 B - k} = \frac{2k}{B}$. If $B > 4k$, we have $\frac{k}{(1-p_2)B} = \frac{k}{B-2k} < 1/2$, and $\frac{p_2 k}{p_2 B - k} = \frac{2k}{B} < 1/2$. Hence, if $p_2 = \frac{2k}{B}$, then any $p_1 \in (1/2, 1)$ satisfies the required conditions. Moreover, it is easy to see that around any (p_1, p_2) such that $p_1 \in (1/2, 1)$ and $p_2 = \frac{2k}{B}$ there is an open ball that satisfies the required conditions.

□

3 Comparing Institutions cont'd (Proof of Proposition 3)

Proposition 3 (***Selection***). *1. Selection is strictly better under bundling when effort under bundling is positive and, in each policy area, weakly higher than under unbundling.*

2. Selection is strictly better under unbundling only if there is at least one policy area in which the effort under unbundling is strictly higher than the effort under bundling.

Proof of Proposition 3. Denote by $U_P^U(a_1, a_2)$ the ex ante post-election welfare of the Principal under unbundling when (a_1, a_2) is exerted in equilibrium. We have

$$U_P^U(a_1 = 1, a_2 = 1) = \sum_{i=1}^{2} \left[\pi \left[p_i^H R + (1 - p_i^H)\pi R\right] + (1 - \pi)(1 - p_i^L)\pi R\right]$$

$$U_P^U(a_i = 1, a_j = 0) = \pi \left[p_i^H R + (1 - p_i^H)\pi R\right] + (1 - \pi)(1 - p_i^L)\pi R + \pi R$$

$$U_P^U(a_1 = 0, a_2 = 0) = 2\pi R.$$

Simple algebra establishes that $U_P^U(a_1 = 1, a_2 = 1) > U_P^U(a_i = 1, a_j = 0) > U_P^U(a_1 = 0, a_2 = 0)$.

Similarly, denote $U_P^B((a_1, a_2), r)$ the ex ante post-election welfare of the Principal under bundling when (a_1, a_2) is exerted by the Agent and the Principal uses retention rule r.

We have

$$
\begin{aligned}
U_P^B(a_1 = 1, a_2 = 1, r_s) &= \pi\left[p_1^H p_2^H 2R + (1 - p_1^H p_2^H)\pi 2R\right] + (1-\pi)(1 - p_1^L p_2^L)\pi 2R \\
U_P^B(a_1 = 1, a_2 = 1, r_m) &= \pi\left[(p_1^H + p_2^H - p_1^H p_2^H)2R + (1 - p_1^H)(1 - p_2^H)\pi 2R\right] \\
&\quad + (1-\pi)(1 - p_1^L)(1 - p_2^L)\pi 2R \\
U_P^B(a_i = 1, a_j = 0, r_i) &= \pi\left[p_i^H 2R + (1 - p_i^H)\pi 2R\right] + (1-\pi)(1 - p_i^L)\pi 2R \\
U_P^U(a_1 = 0, a_2 = 0, r) &= 2\pi R.
\end{aligned}
$$

Simple algebra establishes (1) that $U_P^B(a_i = 1, a_j = 0, r_i) > U_P^B(a_1 = 0, a_2 = 0, r)$, (2) that $U_P^B(a_1 = 1, a_2 = 1, r_m) > U_P^B(a_i = 1, a_j = 0, r_i)$ when $p_i^H(1 - p_j^H) \geq p_i^L(1 - p_j^L)$ for all $i = 1, 2, j \neq i$, and (3) that $U_P^B(a_1 = 1, a_2 = 1, r_s) > U_P^B(a_i = 1, a_j = 0, r_i)$ when $p_i^H(1 - p_j^H) \leq p_i^L(1 - p_j^L)$ for all $i = 1, 2, j \neq i$.

Some more algebra then establishes (1) that $U_P^B(a_1 = 1, a_2 = 1, r_m) > U_P^U(a_1 = 1, a_2 = 1)$ when $p_i^H(1 - p_j^H) \geq p_i^L(1 - p_j^L)$ for all $i = 1, 2, j \neq i$, (2) $U_P^B(a_1 = 1, a_2 = 1, r_s) > U_P^U(a_1 = 1, a_2 = 1)$ when $p_i^H(1 - p_j^H) \leq p_i^L(1 - p_j^L)$ for all $i = 1, 2, j \neq i$, and (3) that $U_P^B(a_i = 1, a_j = 0, r_i) > U_P^U(a_i = 1, a_j = 0)$, which, combined with $U_P^U(a_1 = 1, a_2 = 1) > U_P^U(a_i = 1, a_j = 0) > U_P^U(a_1 = 0, a_2 = 0)$, establishes Part 1 of Proposition 3.

Finally, we have (1) $U_P^U(a_1 = 1, a_2 = 1) > U_P^B(a_i = 1, a_j = 0, r_i)$ if, and only if, $p_i^H - p_i^L < p_j^H - p_j^L$, (2) $U_P^U(a_1 = 1, a_2 = 1) > U_P^B(a_1 = 0, a_2 = 0, r)$, and (3) $U_P^U(a_i = 1, a_j = 0) > U_P^B(a_1 = 0, a_2 = 0, r) = U_P^U(a_1 = 0, a_2 = 0)$, which establishes Part 2 of Proposition 3. □

4 Comparing Institutions cont'd (Proof of Proposition 4)

Proposition 4. *1. Bundling supports higher equilibrium welfare than unbundling under conditions (1) and (2) in Proposition 1.*

2. Comparing across policy-making environments, the highest levels of accountability, as measured by the maximally effective incentives and selection, occur under bundling.

Proof of Proposition 4. Follows directly from Propositions 1, 2, and 3. □

5 Influence of Special Interests (Proof of Proposition 5)

Proposition 5. *Suppose that in the absence of IG, the Agent(s) expend effort in both policy areas under bundling and under unbundling. Then the arrival of IG (weakly) strengthens the Principal's marginal preference for bundling when, in equilibrium, the policy-making environment supports effort in both policy areas under the moderate retention rule and the complexity of policy area 2 is sufficiently high ($p_2 < \sqrt{k}/\sqrt{B}$), and (weakly) weakens it otherwise.*

To prove Proposition 5, we proceed in the following steps. We first characterize, in Proposition A. 1, the equilibrium levels of effort of the Agent under bundling in the presence of IG, using the bribe levels described in Lemma A. 4 as a stepping stone. We then characterize the equilibrium levels of effort of the Agents under unbundling in the presence of IG in Proposition A. 2. Using these results, we finally prove Proposition 5.

Lemma A. 4. *Under bundling:*

1. *Suppose the Principal retains the Agent if, and only if, there is success on both tasks and assume that* $p_1p_2B - 2k \geq 0$*. Then, IG offers the Agent a bribe* $b = p_1p_2B - 2k$ *if, and only if,* $u_{IG} \geq p_2B - 2k/p_1$.

2. *Suppose the Principal retains the Agent if, and only if, there is success on at least one task and assume that* $p_i(1-p_j)B - k \geq 0$ *for all* $i = 1, 2$*. Then, IG offers the Agent a bribe* $b = p_1(1-p_2)B - k$ *if, and only if,* $u_{IG} \geq (1-p_2)B - k/p_1 =: \check{u}(p_1, p_2)$.

Proof of lemma A. 4.

1. Suppose the Principal retains the Agent if, and only if, there is success on both tasks and assume that $p_1p_2B - 2k \geq 0$. Then, if IG does not offer a bribe, the Agent chooses $(a_1 = 1, a_2 = 1)$. The Agent's expected utility is then $p_1p_2B - 2k$. If IG offers the Agent a bribe b and the Agent accepts the bribe, the Agent chooses $(a_1 = 0, a_2 = 0)$ and receives an expected utility of b. It follows that the Agent accepts the bribe b if, and only if, $b \geq p_1p_2B - 2k$. IG thus chooses between the lowest bribe that the Agent accepts, i.e. $b = p_1p_2B - 2k$ and $b = 0$. Upon offering $b = p_1p_2B - 2k$, IG receives a payoff of $u_{IG} - b = u_{IG} - (p_1p_2B - 2k)$. Upon offering $b = 0$, IG receives a payoff of $(1-p_1)u_{IG}$. Hence, IG offers $b = p_1p_2B - 2k$ if, and only if, $u_{IG} \geq p_2B - 2k/p_1$.

2. Suppose the Principal retains the Agent if, and only if, there is success on at least one task and assume that $p_i(1-p_j)B - k \geq 0$ for all $i = 1, 2, j \neq i$. Then, if IG does not offer a bribe, the Agent chooses $(a_1 = 1, a_2 = 1)$. The Agent's expected utility is then $(p_1p_2 + p_1(1-p_2) + (1-p_1)p_2)B - 2k$. Note that $p_2(1-p_1)B - k \geq 0$ implies $p_2B - k \geq 0$. Hence, if IG offers the Agent a bribe b and the Agent accepts the bribe, the Agent chooses $(a_1 = 0, a_2 = 1)$ and receives an expected utility of $b + p_2B - k$. It follows that the Agent accepts the bribe b if, and only if, $b + p_2B - k \geq (p_1p_2 + p_1(1-p_2) + (1-p_1)p_2)B - 2k$, i.e. if, and only if, $b \geq p_1(1-p_2)B - k$. IG thus chooses between the lowest bribe that the Agent accepts, i.e. $b = p_1(1-p_2)B - k$ and $b = 0$. Upon offering

$b = p_1(1-p_2)B - k$, IG receives a payoff of $u_{IG} - b = u_{IG} - (p_1(1-p_2)B - k)$. Upon offering $b = 0$, IG receives a payoff of $(1-p_1)u_{IG}$. Hence, IG offers $b = p_1(1-p_2)B - k$ if, and only if, $u_{IG} \geq (1-p_2)B - k/p_1$.

□

Proposition A. 1. *On the equilibrium path of play under bundling, the Agent chooses to exert effort in both policy areas if, and only if, the pair of policy-making environment and beliefs satisfies either of the following two sets of conditions:*

1. *(a) the complexity of each task, as well as the value of failure to IG, are sufficiently low,* $p_1 \geq \frac{2k}{p_2 B - u_{IG}}$*;*

   (b) the Principal expects the Agent to exert effort in both policy areas; and

   (c) the Principal's estimation of the Agent's competence decreases unless the outcome is success on both tasks, $p_i^H(1-p_j^H) \leq p_i^L(1-p_j^L)$ *for all* $i = 1, 2$*;*

   or

2. *(a) the complexity of each task is moderate and the value of failure to IG is sufficiently low,* $1 - \frac{k}{p_2 B} \geq p_i \geq \frac{k}{(1-p_2)B - u_{IG}}$*; and*

   (b) the Principal's estimation of the Agent's competence increases when the outcome is success on at least one task, $p_1^H(1-p_2^H) \geq p_1^L(1-p_2^L)$*, and* $p_2^H(1-p_1^H F(\check{u}(p_1,p_2))) \geq p_2^L(1-p_1^L F(\check{u}(p_1,p_2)))$.

When conditions (1) hold, the Principal adopts the strict retention rule, when conditions (2) hold, the moderate retention rule.

Proof of Proposition A. 1. 1. From Proposition 1 we know that the Agent chooses to exert effort on both tasks if, and only if, the Principal uses either the strict retention rule or the moderate retention rule.

(a) Suppose the Principal uses the strict retention rule. The Agent then chooses to exert effort on both tasks if, and only if, $p_1 p_2 B - 2k \geq 0$ and IG did not bribe the Agent. From Lemma A. 4, IG, in turn, does not bribe the Agent if, and only if, $u_{IG} \leq p_2 B - 2k/p_1$ which is equivalent to $p_1 \geq \frac{2k}{p_2 B - u_{IG}} \geq \frac{2k}{p_2 B}$. The conditions under which it is sequentially rational for the Principal to use the strict retention rule are not altered by the possibility of IG influence. The derivation is similar to the one found in the proof of Lemma A. 1.

(b) Suppose the Principal uses the moderate retention rule. The Agent then chooses to exert effort on both tasks if, and only if, $1 - \frac{k}{p_2 B} \geq p_1 \geq \frac{k}{(1-p_2)B}$, and IG did not bribe the Agent. From Lemma A. 4, IG, in turn, does not bribe the Agent if, and only if, $u_{IG} \leq (1 - p_2)B - k/p_1 = \check{u}(p_1, p_2)$ which is equivalent to $p_1 \geq \frac{k}{(1-p_2)B - u_{IG}} \geq \frac{k}{(1-p_2)B}$. When IG bribes the Agent, the Agent chooses $(a_1 = 0, a_2 = 1)$ if, and only if, $p_2 B - k \geq 0$. Note that $1 - \frac{k}{p_2 B} \geq p_1 \geq \frac{k}{(1-p_2)B}$ implies $p_2 B - k \geq 0$.

We now derive the conditions under which it is sequentially rational for the Principal to use the moderate retention rule given that the Principal believes that the Agent and IG are best-responding to such a retention strategy. To understand the construction of the beliefs, remember that the Principal is uncertain about the value u_{IG} that IG attaches to policy failure and that u_{IG} is drawn from a distribution function $F(\cdot)$ with full support on the non-negative real line $\mathbb{R}_+$. It follows that the Principal expects the following action profile: with probability $F(\check{u}(p_1, p_2)) \in (0, 1)$ IG does not to bribe the Agent who chooses $(a_1 = 1, a_2 = 1)$, while with probability $(1 - F(\check{u}(p_1, p_2)))$ IG bribes the Agent who then chooses $(a_1 = 0, a_2 = 1)$. We denote $(\mathring{a}_1^F, \mathring{a}_2 = 1)$ these expectations of the Principal about the Agent's actions. We thus have

$$\begin{aligned}
Pr(\theta &= \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) = \frac{F(\check{u}(p_1,p_2))p_1^H p_2^H \pi}{F(\check{u}(p_1,p_2))p_1^H p_2^H \pi + F(\check{u}(p_1,p_2))p_1^L p_2^L \pi}, \\
Pr(\theta &= \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \\
&= \frac{\left[(1-p_2^H)(1-p_1^H F(\check{u}(p_1,p_2)))\right]\pi}{\left[(1-p_2^H)(1-p_1^H F(\check{u}(p_1,p_2)))\right]\pi + \left[(1-p_2^L)(1-p_1^L F(\check{u}(p_1,p_2)))\right](1-\pi)}, \\
Pr(\theta &= \theta_H|(o_1 = s, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \\
&= \frac{F(\check{u}(p_1,p_2))p_1^H(1-p_2^H)\pi}{F(\check{u}(p_1,p_2))p_1^H(1-p_2^H)\pi + F(\check{u}(p_1,p_2))p_1^L(1-p_2^L)\pi}, \\
Pr(\theta &= \theta_H|(o_1 = f, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \\
&= \frac{\left[p_2^H - F(\check{u}(p_1,p_2))p_1^H p_2^H\right]\pi}{\left[p_2^H - F(\check{u}(p_1,p_2))p_1^H p_2^H\right]\pi + \left[p_2^L - F(\check{u}(p_1,p_2))p_1^L p_2^L\right](1-\pi)}.
\end{aligned}$$

As in the baseline model, the Principal updates favorably on the type of the Agent upon observing success on both tasks, and negatively upon observing failure on both tasks. Indeed, we have $Pr(\theta = \theta_H|(o_1 = s, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) > \pi$ and $Pr(\theta = \theta_H|(o_1 = f, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) < \pi$, because $p_i^H > p_i^L$ for all $i = 1, 2$. But, the conditions under which the Principal updates favorably upon observing success on one task and failure on another differ from those of the baseline model. While, as in the baseline model, $Pr(\theta = \theta_H|(o_1 = s, o_2 = f); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \geq \pi$ if, and only if, $p_1^H(1-p_2^H) \geq p_1^L(1-p_2^L)$, we have $Pr(\theta = \theta_H|(o_1 = f, o_2 = s); (\mathring{a}_1^F, \mathring{a}_2 = 1)) \geq \pi$ if, and only if, $p_2^H(1 - p_1^H F(\check{u}(p_1,p_2))) \geq p_1^L(1 - p_2^L F(\check{u}(p_1,p_2)))$.

□

Proposition A. 2. *For all (p_i, B_i, k) such that $p_i B_i - k \geq 0$,*

1. *the Interest Group offers a bribe $b_u = p_1 B_1 - k$ if, and only if, $u_{IG} \geq B_1 - k/p_1 \geq 0$;*

2. *Agent A_2 chooses to exert effort, and Agent A_1 accepts the bribe if, and only if, $b \geq p_1 B_1 - k$, and chooses $a_1 = 0$ if accepting and $a_1 = 1$ if rejecting; and*

3. *the Principal retains Agent A_i if, and only if, the outcome is success on task i.*

For all (p_i, B_i, k) such that $p_iB_i - k < 0$, Agent A_i chooses to exert no effort on task i independent of the bribe and the Principal's retention rule.

Proof of Proposition A. 2. Suppose the Principal reelects A_i if, and only if, $o_i = s$. The expected payoff to Agent A_i of choosing to exert effort is then $p_iB_i - k$. Thus, if $p_1B_1 - k < 0$, Agent A_1 will choose not to exert effort independent of the offer from IG, which, therefore, has no incentive to influence A_1's behavior. If, on the other hand, $p_1B_1 - k \geq 0$, A_1 exerts effort absent a bribe and IG has an incentive to influence him. In this case, A_1 accepts a bribe if, and only if, $b \geq p_1B_1 - k$. If IG offers A_1 b_u, IG receives a payoff of $u_{IG} - b_u = u_{IG} - p_1B_1 + k$. If IG offers A_1 $b = 0$, IG receives an expected payoff of $(1 - p_1)u_{IG}$. Hence, IG offers b_u if, and only if, $u_{IG} \geq B_1 - k/p_1$. Moreover, A_2 chooses $a_2 = 1$ if, and only if, $p_2B_2 - k \geq 0$. It is obvious that the Principal's retention rule is sequentially rational whenever $p_iB_i - k \geq 0$. □

Proof of Proposition 5. We first show that for any vector of parameter values for which, in the absence of IG, the Agent exerts effort on both tasks under bundling with the Principal using the strict retention rule (condition 1.(a) in Proposition 1), if the presence of IG makes a difference for the Agent(s)'s choices, then it creates a strict incentive advantage for unbundling over bundling and never creates a strict incentive advantage for bundling. In a second step, we show that if the vector of parameter values sustains an equilibrium under bundling in which, in the absence of IG, the Agent exerts effort on both tasks under bundling with the Principal using the moderate retention, then the presence of IG can create a strict incentive advantage for bundling over unbundling.

1. Following from Proposition A. 2, we know that if $p_1 \geq \frac{p_2k}{p_2(B-u_{IG})-k}$, there exists (B_1, B_2) such that effort on both tasks can be sustained under unbundling. We now show that if $B > 2k + u_{IG}$, we have $\frac{2k}{p_2B-u_{IG}} > \frac{p_2k}{p_2(B-u_{IG})-k}$ for all $p_2 \in \left[\frac{2k+u_{IG}}{B}, 1\right]$. Note that $\frac{2k}{p_2B-u_{IG}} > \frac{p_2k}{p_2(B-u_{IG})-k}$ if, and only if, $Q(p_2) := p_2^2B - p_2(2B - u_{IG}) + 2k < 0$. $\frac{dQ}{dp_2} = 2p_2B - (2B - u_{IG})$. Hence, $Q(p_2)$ is decreasing on $\left[0, 1 - \frac{u_{IG}}{2B}\right]$ and increasing

on $\left[1-\frac{u_{IG}}{2B},1\right]$. It follows that the maximum of $Q(p_2)$ on $\left[\frac{2k+u_IG}{B},1\right]$ is reached at $p_2=\frac{2k+u_{IG}}{B}$ or at $p_2=1$. Simple algebra shows that, if $B>2k+u_{IG}$, then $Q(p_2=\frac{2k+u_{IG}}{B}), Q(p_2=1)<0$. It follows that there does not exist $(p_1^H,p_1^L,p_2^H,p_2^L,\pi,k,B,u_{IG})$ such that effort is exerted on both tasks under bundling and strict retention rule, yet effort is not exerted on both tasks under unbundling. In other words, for any parameter vector that sustains, in the absence of IG, an equilibrium in which the Agent exerts effort on both tasks with the Principal using the strict retention rule, and for any $u_{IG}\geq 0$, unbundling has a weak incentive advantage over bundling in the presence of IG.

Now choose any vector of parameter values $\boldsymbol{s}\in\boldsymbol{\Sigma}$ that satisfies condition 1.(a) in Proposition 1. This requires, in particular, that $1\geq p_1\geq\frac{2k}{p_2B}$. Note that we established in the proof of Proposition 2 that $\frac{2k}{p_2B}>\frac{p_2k}{p_2B-k}$ for all $p_2\in\left[\frac{2k}{B},1\right]$. Hence, there exists (B_1,B_2) such that effort is exerted on both tasks under unbundling for any such $\boldsymbol{s}\in\boldsymbol{\Sigma}$. It is easy to see that there exist $\underline{u}_{IG}(p_1,p_2),\overline{u}_{IG}(p_1,p_2)\in[0,p_2B)$ such that $1\geq p_1=\frac{2k}{p_2B-\underline{u}_{IG}(p_1,p_2)}$ and $1\geq p_1=\frac{p_2k}{p_2(B-\overline{u}_{IG}(p_1,p_2))-k}$. Because, whenever $B>2k+u_{IG}$, $\frac{2k}{p_2B-u_{IG}}>\frac{p_2k}{p_2(B-u_{IG})-k}$ for all $p_2\in\left[\frac{2k+u_{IG}}{B},1\right]$, we have $\overline{u}_{IG}(p_1,p_2)>\underline{u}_{IG}(p_1,p_2)$. Moreover, as $\frac{2k}{p_2B-u_{IG}}$ is increasing on $[0,p_2B]$, we have $p_1<\frac{2k}{p_2B-u_{IG}}$ for all $u_{IG}>\underline{u}_{IG}(p_1,p_2)$, which implies that the Agent does not exert effort on both tasks under bundling when $u_{IG}>\underline{u}_{IG}(p_1,p_2)$. Similarly, as $\frac{p_2k}{p_2(B-u_{IG})-k}$ is increasing in u_{IG}, we have $p_1\geq\frac{p_2k}{p_2(B-u_{IG})-k}$ for all $u_{IG}\leq\overline{u}_{IG}(p_1,p_2)$ and effort is exerted on both tasks under unbundling. Thus, for any $\boldsymbol{s}\in\boldsymbol{\Sigma}$ that satisfies condition 1.(a) in Proposition 1, there exists u_{IG} such that unbundling has a strict incentive advantage over bundling.

2. Consider now any vector of parameter values $\boldsymbol{s}\in\boldsymbol{\Sigma}$ that sustains effort on both tasks under bundling and under unbundling with the Principal using the moderate retention rule under bundling (condition 1.(b) in Proposition 1). Then, by Proposition A. 1 if

the Agent is bribed under bundling he exerts effort on task 2. Hence, for the expected effort to be (weakly) higher on task 2 under unbundling, we need $p_2B_2 - k \geq 0$, or equivalently $B_2 \geq k/p_2$. By Proposition A. 2 the bribe that IG needs to pay to A_1 to contract failure is $b_u = p_1B_1 - k$. The feasible pair (B_1, B_2) that maximizes b_u, while sustaining $a_2 = 1$ under unbundling is thus $(B_1 = B - k/p_2, B_2 = k/p_2)$. We then have $b_u = p_1(B - k/p_2) - k$. If the Principal uses the moderate retention rule, we have $b = p_1(1 - p_2)B - k$ (see Lemma A. 4). Simple algebra establishes that $b \geq b_u$ if, and only if, $p_2 \leq \frac{\sqrt{k}}{\sqrt{B}}$. Suppose $p_2 < \frac{\sqrt{k}}{\sqrt{B}}$. Then, if $B_2 \geq k/p_2$ IG bribes the Agent either (1) under neither institution, or (2) under both, or (3) under unbundling but not under bundling. If $B_2 < k/p_2$, Agent A_2 does not exert effort on task 2 under unbundling. Hence, for any vector of parameter values $\boldsymbol{s} \in \boldsymbol{\Sigma}$ that sustains effort on both tasks under bundling and under unbundling with the Principal using the moderate retention rule under bundling (condition 1.(b) in Proposition 1), if $p_2 < \frac{\sqrt{k}}{\sqrt{B}}$, then the presence of IG can create a strict incentive advantage of bundling.

□

6 Transparency of Actions

So far, we have assumed that the Principal does not observe the effort choices of the Agent. We refer to that case as the case with "no transparency." In this section we consider the implications of having the Principal observe the actions a_1, a_2, chosen by the Agent(s) (the case with "transparency") and show that the incentives through which bundling comes to dominate unbundling still operate under transparency of actions, but now the insurance mechanism that underlies effort investment into both policy areas under the moderate retention rule becomes even more essential to the priority of bundling. We have the following result:

Proposition 6. *Introducing transparency*

1. *eliminates the possibility of an equilibrium in which the Agent exerts effort on both tasks and the Principal uses the strict retention rule and (weakly) decreases the relative benefits of bundling overall; but*

2. *has no effect on the existence of the equilibrium in which the Agent exerts effort on both tasks and the Principal uses the moderate retention rule and so leaves bundling as the superior institution when the policy area complexities are sufficiently moderate and sufficiently symmetric.*

To prove Proposition 6, we first provide, in Proposition A. 3, a characterization of the conditions under which the Agent exerts effort in both areas under bundling with transparency of actions.

Proposition A. 3. *On the equilibrium path of play under bundling with transparency of actions, the Agent chooses to exert effort in both policy areas if, and only if, the pair of policy-making environment and beliefs satisfies either of the following set of conditions:*

(a) the complexity of each task is moderate, $1 - \frac{k}{p_j B} \geq p_i \geq \frac{k}{(1-p_j)B}$*; and*

(b) the Principal's estimation of the Agent's competence increases when the outcome is success on at least one task, $p_i^H(1-p_j^H) \geq p_i^L(1-p_j^L)$ *for all* $i = 1, 2$.

When these conditions hold the Principal adopts the moderate retention rule.

Proof of Proposition A. 3. Suppose the Agent chooses $(a_i = 1, a_j = 0)$. By Lemma A. 1, the Principal then retains the Agent if $(o_i = s, o_J = f)$ and does not retain if $(o_i = f, o_j = f)$. Consequently, the Agent's expected utility from choosing $(a_i = 1, a_j = 0)$ is $p_i B - k$.

Suppose next the Agent chooses $(a_1 = 1, a_2 = 1)$. By Lemma A. 1, the Principal then uses one of the following three retention rules:

1. If $p_i^H(1-p_j^H) < p_i^L(1-p_j^L)$ for all $i = 1, 2$, the Principal retains if, and only if $(o_1 = s, o_2 = s)$.

2. If $p_i^H(1-p_j^H) > p_i^L(1-p_j^L)$ for all $i = 1, 2$, the Principal retains if, and only if $o_i = s$ for at least some task $i = 1, 2$.

3. If $p_i^H(1-p_j^H) > p_i^L(1-p_j^L)$ for $i = 1, 2$, and $p_j^H(1-p_i^H) < p_j^L(1-p_i^L)$ for $j \neq i$, the Principal retains if, and only if $o_i = s$ for $i = 1, 2$.

In case 1. the Agent's expected utility from choosing $(a_1 = 1, a_2 = 1)$ is $p_1 p_2 B - 2k$. In case 2. the Agent's expected utility from choosing $(a_1 = 1, a_2 = 1)$ is $(p_1 + p_2 - p_1 p_2)B - 2k$. In case 3. the Agent's expected utility from choosing $(a_1 = 1, a_2 = 1)$ is $p_i B - 2k$. Finally choosing $(a_1 = 0, a_2 = 0)$ induces, by Lemma A. 1 again, the Principal to dismiss the Agent who then receives a payoff of 0.

The Agent then chooses the effort allocation (a_1, a_2) which maximizes his expected utility given the retention rule that is induced by (a_1, a_2). Because $p_1 p_2 B - 2k < p_i B - k$ and $p_i B - 2k < p_i B - k$ for all $i = 1, 2$, the Agent never chooses $(a_1 = 1, a_2 = 1)$ if it induces retention rule 1. or 3. The Agent chooses $(a_1 = 1, a_2 = 1)$ if, and only if, $(a_1 = 1, a_2 = 1)$ induces retention rule 2. and $(p_1 + p_2 - p_1 p_2)B - 2k \geq p_i B - k \geq 0$ for all $i = 1, 2$, which is equivalent to $p_i(1 - p_j)B - k \geq 0$ for all $i = 1, 2$. The Agent chooses $(a_i = 1, a_j = 0)$ if, and only if, one of the two following sets of conditions hold: (1) $(a_1 = 1, a_2 = 1)$ induces retention rule 2. and $p_i B - k \geq \max\{p_j B - k, (p_1 + p_2 - p_1 p_2)B - 2k, 0\}$, or (2) $(a_1 = 1, a_2 = 1)$ does not induce retention rule 2. and $p_i B - k \geq \max\{p_j B - k, 0\}$. The Agent chooses $(a_1 = 0, a_2 = 0)$ if, and only if, $0 > p_i B - k$ for all $i = 1, 2$. Note that $0 > p_i B - k$ for all $i = 1, 2$, implies $0 > (p_1 + p_2 - p_1 p_2)B - 2k$. □

Second, we note that transparency does not affect the positive effort equilibrium under unbundling: even when the Principal observes that the Agent exerted effort, the Principal retains the Agent if, and only if, the outcome is success, as she wants to retain a high competence Agent and, as before, success is a signal of high competence and failure a signal of low competence. Consequently, the range of parameter values that sustain an equilibrium in which both Agents exert effort is not altered by transparency of actions.

Proof of Proposition 6. Follows directly from Propositions 1, 2, 3, A. 3, and the fact that transparency does not affect equilibrium play under unbundling. □

7 Interactions between Tasks

In this section, we consider two ways in which we may see spillovers across policy areas: with respect to (1) costs of effort, and (2) the probabilities of success. We show that our key qualitative conclusions about when accountability under bundling is better and when it is worse hold in cases (1) and (2) as long as the spillover effects are not too negative; when they are, unbundling provides better incentives than bundling.

To study the first case, we will assume that the cost of exerting effort in policy area $i = 1, 2$, is $k > 0$ if no effort is exerted in area $j \neq i$, but γk if effort is exerted in area $j \neq i$. The cost of exerting effort in both areas under bundling, will, consequently, become $2\gamma k$. We impose the restriction that exerting effort in both areas is more costly than exerting effort in a single area, i.e. we assume $2\gamma k > k$, or equivalently $\gamma > 1/2$. If $\gamma \in (1/2, 1)$, we say that there are positive spillover effect between both policy areas: exerting effort in area $i = 1, 2$, reduces the cost of effort in area $j \neq i$. If $\gamma > 1$, there are negative spillover effects between areas: exerting effort in area $i = 1, 2$, increases the cost of effort in area $j \neq i$.

We have the following result:

Proposition 7. *1. If $\gamma \in (\frac{1}{2}, \frac{B}{8k} + \frac{1}{2})$, then there exist vectors of parameters $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ for which bundling has a strict incentive advantage over unbundling and the Principal uses the moderate retention rule.*

2. *If* $\gamma \in (\frac{B}{8k}+\frac{1}{2}, \frac{B}{2k})$ *then there exist vectors of parameters* $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ *for which unbundling has a strict incentive advantage over bundling, but no vectors* $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ *for which bundling has a strict incentive advantage.*

Part 1 of Proposition 7 states that bundling can have a strict incentive advantage over unbundling, which will occur under the moderate retention rule even when there are negative spillover effects between both areas. When the value of holding office is high relative to the cost of effort, i.e. $B > 4k$, we have $\frac{B}{8k} + \frac{1}{2} > 1$ and bundling may dominate unbundling in terms of incentives even if $\gamma > 1$. This shows that the argument in favor of bundling is robust to the possibility of interactions between policy areas. Part 2 of Proposition 7 states that the negative spillover effects cannot be too strong, however: if $\gamma > \frac{B}{8k} + \frac{1}{2}$, bundling can never generate a strict incentive advantage over unbundling.

The intuition behind this result is simple. Under the moderate retention rule, conditional on exerting effort in area i, the Agent chooses to exert effort in area j, only if the insurance benefit of doing so, namely $p_j(1 - p_i)B$, exceeds the cost γk of exerting effort in area j. When γ is very high, exerting effort only in area i still provides the Agent with a chance at retention and moreover reduces the cost of effort in area i from γk to k. The single Agent thus coordinates her effort choices to avoid having to pay the costs of exerting effort in both areas when the spillover effects between both areas are strong. Under unbundling such a coordination is unavailable. Each Agent A_i has no other choice but to exert effort in area i in order to be retained. Consequently, effort in both areas can be sustained under unbundling for levels of negative spillovers γ for which the single Agent under bundling would choose to exert effort in only one area rather than in both.

Proof of Proposition 7. 1. Proceeding as in the paper, we find that there exists a feasible pair (B_1, B_2) such that effort is exerted in both policy areas under unbundling if, and only if, $p_1 \geq \frac{p_2\gamma k}{p_2 B - \gamma k}$. Let p_2^U solve $p_2 = \frac{p_2\gamma k}{p_2 B - \gamma k}$. We have $p_2^U = \frac{2\gamma k}{B}$. For all

$p_2 < p_2^U$, we have $p_2 < \frac{p_2 \gamma k}{p_2 B - \gamma k}$. Therefore, for all (p_1, p_2) such that $p_1 = p_2$ and $p_2 < p_2^U$, $(a_1 = 1, a_2 = 1)$ cannot be sustained in equilibrium under unbundling.

Similarly, proceeding as in the proof of Proposition 1, we find that for an equilibrium to exist in which the Agent exerts effort in both areas and the Principal uses the moderate retention rule it must be the case that $p_i(1-p_j)B-(2\gamma-1)k \geq 0$ for all $i = 1, 2, j \neq i$, which is equivalent to $1 - \frac{k(2\gamma-1)}{p_2 B} = \frac{k(2\gamma-1)}{(1-p_2)B}$. Now let $\underline{p}_2$ and $\overline{p}_2$ be the solutions to $1 - \frac{k(2\gamma-1)}{p_2 B} = \frac{k(2\gamma-1)}{(1-p_2)B - u_{IG}}$. We have

$$\underline{p}_2 = \frac{1}{2} - \frac{\sqrt{B}\sqrt{B - 4k(2\gamma-1)}}{2B}$$

and

$$\overline{p}_2 = \frac{1}{2} + \frac{\sqrt{B}\sqrt{B - 4k(2\gamma-1)}}{2B}.$$

Notice that if $\gamma < \frac{B}{8k} + \frac{1}{2}$, then $\underline{p}_2$ and $\overline{p}_2$ are real numbers with $\underline{p}_2 < \overline{p}_2$. Simple algebra also establishes that $\underline{p}_2$ and $\overline{p}_2$ also solve $p_2 = 1 - \frac{k(2\gamma-1)}{p_2 B}$ and, consequently, $p_2 = \frac{k(2\gamma-1)}{(1-p_2)B}$ and that $1 - \frac{k(2\gamma-1)}{p_2 B} > p_2 > \frac{k(2\gamma-1)}{(1-p_2)B}$ for all $p_2 \in (\underline{p}_2, \overline{p}_2)$. Hence, for all (p_1, p_2) such that $p_1 = p_2$ and $p_2 \in \left[\underline{p}_2, \overline{p}_2\right]$, there exists, following arguments given in the proof of Proposition 2, infinitely many vectors $(p_1^H, p_1^L, p_2^H, p_2^L, \pi)$ for which $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under bundling with the Principal using the moderate retention rule.

Simple algebra shows that if $\gamma = 1$ and $B > 4k$, then $\underline{p}_2 < p_2^U$. It follows that if $\gamma = 1$ and $B > 4k$, there exists infinitely many vectors (p_1, p_2) such that $p_1 = p_2$ and $p_2 \in \left[\underline{p}_2, p_2^U\right)$, and for which, therefore, $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under bundling, but not under unbundling. Now note that $\underline{p}_2$ and $\overline{p}_2$ are well defined, continuous functions of γ if, and only if, $B - 4k(2\gamma - 1) > 0$, i.e. if, and only if, $\gamma < \frac{B}{8k} + \frac{1}{2}$. Moreover, if $B > 4k$, we have $\frac{B}{8k} + \frac{1}{2} > 1$. p_2^U is also a continuous function of γ. By continuity, there exists $\overline{\gamma} > 1$, such that for all $\gamma < \overline{\gamma}$, $\underline{p}_2 < \overline{p}_2$ and $\underline{p}_2 < p_2^U$, which establishes the result.

2. Proceeding as in the proof of Proposition 1, we find that for an equilibrium to exist in which the Agent exerts effort in both policy areas and the Principal uses the strict retention rule, it must be the case that $p_1 \geq \frac{2\gamma k}{p_2 B}$. Remember also that for an equilibrium to exist in which the Agent exerts effort in both policy areas and the Principal uses the moderate retention rule, it must be the case that $1 - \frac{k(2\gamma-1)}{p_2 B} \geq p_1 \geq \frac{k(2\gamma-1)}{(1-p_2)B}$. As shown in part 1. of this proof if $\gamma > \frac{B}{8k} + \frac{1}{2}$ the second case cannot occur. In turn, $(a_1 = 1, a_2 = 1)$ can be sustained in equilibrium under unbundling if, and only if, $1 \geq p_1 \geq \frac{p_2\gamma k}{p_2 B - \gamma k}$. Notice that $p_1 \geq \frac{2\gamma k}{p_2 B}$ and $1 \geq p_1 \geq \frac{p_2 \gamma k}{p_2 B - \gamma k}$ can be satisfied if, and only if, $\gamma < \frac{B}{2k}$. Proceeding as in the proof of Proposition 2, we can show that for all values of (p_1, p_2) such that $1 \geq p_1 \geq \frac{2\gamma k}{p_2 B}$, we have $\frac{p_2\gamma k}{p_2 B - \gamma k} < \frac{2\gamma k}{p_2 B}$ and therefore $1 \geq p_1 > \frac{p_2 \gamma k}{p_2 B - \gamma k}$. Finally, notice that if $B > 2k$, we have $\frac{B}{8k} + \frac{1}{2} < \frac{B}{2k}$.

□

So far, we have considered the case where tasks interact through the cost factor: exerting effort on task i increases or decreases the cost of effort on task $j \neq i$. We now consider the case where exerting effort on task i affects the probability of success on task $j \neq i$. Formally, we assume that if the Agent exerts effort on task i, then the probability of success on task $j \neq i$ is $\min\{\alpha p_j, 1\}$ where $\alpha > 0$. We will say that there are positive spillovers between tasks if $\alpha \geq 1$, as exerting effort on task $i = 1, 2$, increases the probability of success on task $j \neq i$ conditional on exerting effort on task j. Similarly, We will say that there are negative spillovers between tasks if $\alpha < 1$, as exerting effort on task $i = 1, 2$, then decreases the probability of success on task $j \neq i$ conditional on exerting effort on task j. Modeling interactions between tasks this way yields similar results:

Proposition A. 4. *1. If $\alpha > 2/3$ there exists vectors of parameters $(p_1^H, p_1^L, p_2^H, p_2^L, \pi, B, k)$*

for which bundling has a strict incentive advantage over unbundling and the Principal uses the moderate retention rule.

2. *If $\alpha \leq 2/3$ there exist vectors of parameters $(p_1^H, p_1^L, p_2^H, p_2^L, \pi, B, k)$ for which unbundling has a strict incentive advantage over bundling, but no vectors $(p_1^H, p_1^L, p_2^H, p_2^L, \pi, B, k)$ for which bundling has a strict incentive advantage.*

Proof of Proposition A. 4. Proceeding as in the main body of the article, we start by noting that effort can be sustained on both tasks under unbundling, if, and only if, $B \geq \frac{k}{\alpha p_1} + \frac{k}{\alpha p_2}$.

We now show that if the Agent is incentivized to exert effort on both tasks under bundling via the strict retention rule, then there exists a pair of values of holding office (B_1, B_2) such that effort on both tasks can be sustained under unbundling. Consequently, there are no parameter values for which bundling has a strict incentive advantage over unbundling and the Principal uses the strict retention rule.

To see this, remark that if the Principal uses the strict retention rule then the utility to the Agent of exerting effort on both tasks is $\alpha^2 p_1 p_2 B - 2k$, while not exerting any effort yields a payoff of 0. Proceeding as in the proof of Proposition 1, we find that, if the Principal uses the strict retention rule, then the Agent exerts effort on both tasks under bundling if, and only if, $B \geq \frac{2k}{\alpha^2 p_1 p_2}$. Now note that $\frac{k}{\alpha p_1} + \frac{k}{\alpha p_2} \leq \frac{2k}{\alpha^2 p_1 p_2}$, if, and only if, $\alpha p_1 + \alpha p_2 \leq 2$ which is always true because $\alpha p_i \leq 1$ by properties fo probability functions. Hence, for all values of p_1, p_2, α, k, if $B \geq \frac{2k}{\alpha^2 p_1 p_2}$ then we have $B \geq \frac{k}{\alpha p_1} + \frac{k}{\alpha p_2}$.

We now derive the conditions under which an equilibrium can be sustained under bundling in which the Principal uses the moderate retention rule and the Agent exerts effort on both tasks. Proceeding as in the proof of Proposition 1, we find that the following three conditions need to be satisfied for such an equilibrium to exist:

1. $[\alpha p_j(1 - \alpha p_i) - (1 - \alpha)p_i]\, B - k \geq 0$ for all $i = 1, 2, j \neq i$;

2. $\alpha p_i^H(1-\alpha p_j^H) \geq \alpha p_i^L(1-\alpha p_j^L)$ for all $i = 1, 2, j \neq i$.

Proceeding as in the proof of Proposition 2, let's assume for now that $p_1 = p_2 := p$. Then, if $\frac{k}{\alpha p(1-\alpha p)-(1-\alpha)p} < \frac{2k}{\alpha p}$, there exists, by Lemma A. 3, parameter values for which bundling has a strict incentive advantage over unbundling. We have $\frac{k}{\alpha p(1-\alpha p)-(1-\alpha)p} < \frac{2k}{\alpha p}$, if, and only if, $0 < -2\alpha^2p^2+3\alpha p-2p$, if, and only if, $p < \frac{3}{2\alpha} - \frac{1}{\alpha^2}$. In turn, we have $\frac{3}{2\alpha} - \frac{1}{\alpha^2} > 0$ if, and only if, $\alpha > 2/3$. It follows that if $\alpha > 2/3$, there exists vectors of parameters $(p_1^H, p_1^L, p_2^H, p_2^L, \pi, B, k)$ for which bundling has a strict incentive advantage over unbundling and the Principal uses the moderate retention rule. If $\alpha \leq 2/3$, however, then $\frac{k}{\alpha p(1-\alpha p)-(1-\alpha)p} \geq \frac{2k}{\alpha p}$ and there exist vectors of parameters $(p_1^H, p_1^L, p_2^H, p_2^L, \pi, B, k)$ for which unbundling has a strict incentive advantage over bundling, but no vectors $(p_1^H, p_1^L, p_2^H, p_2^L, \pi, B, k)$ for which bundling has a strict incentive advantage.

□

8 Robustness: Continuous Effort

In the model studied in the paper, we assume that each Agent can choose to exert effort ($a_i = 1$) or not ($a_i = 0$) and that exerting effort produces success with exogenous probability $p_i \in (0, 1)$. In this subsection, we consider the robustness of the arguments in favor of bundling in the context of a version of the model in which the effort choice a_i is continuous and the probability of success p_i is an increasing function of effort. The model is identical to the main model in the paper up to the following modifications: (1) we now assume that $0 < \theta_L < \theta_H$, (2) $a_i \geq 0$ and the cost of effort is a_i^2, and (3) the probability of success is $p_i := c_i(\pi\theta_H + (1-\pi)\theta_L)a_i$, with $c_i > 0$. We interpret the new parameters $c_1, c_2 > 0$ as measures of complexity.

As the purpose of this analysis is to consider the robustness of the arguments with respect to bundling, we suppress the full characterization, and focus on the following result:

Proposition A. 5. *Regardless of action transparency, for any parameter values $B, \pi, \theta_L, \theta_H$, there exists a level of complexity $\tilde{c} \leq \frac{\sqrt{2}}{\sqrt{BT}}$ such that for all $c_1, c_2 < \tilde{c}$,*

1. *in the equilibrium that maximizes the total effort of the Agent under bundling, the Agent exerts positive effort on both tasks and the Principal uses the moderate retention rule;*
2. *under this equilibrium with moderate retention rule, total effort is higher under bundling than under unbundling for any pair (B_1, B_2) of values of holding office.*

Proof of Proposition A. 5. 1. We proceed in a number of steps. We first show that if c_1 and c_2 are sufficiently low, the equilibrium with moderate retention rule is the only equilibrium to sustain positive effort on both tasks. We then show that if c_1 and c_2 are sufficiently low, total effort is higher in the moderate incentives equilibrium than in an equilibrium in which the Agent exerts positive effort only on task i.

We denote $(\mathring{a}_1, \mathring{a}_2)$ the Principal's expectations about the Agent's actions when the Principal does not observe the actions (a_1, a_2) chosen by the Agent. The Principal's posterior belief about the Agent's competence, upon observing outcomes (o_1, o_2) and given expectations $(\mathring{a}_1, \mathring{a}_2)$, is then denoted $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$. Similarly, the Principal's posterior, upon observing outcomes (o_1, o_2) and effort choices (a_1, a_2), is denoted $Pr(\theta = \theta_H | (o_1, o_2); (a_1, a_2))$. Note that we have $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_1, o_2); (a_1, a_2))$ whenever $(o_1, o_2); (\mathring{a}_1, \mathring{a}_2) = (o_1, o_2); (a_1, a_2)$. To simplify notation, we thus only look at $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2))$ in the sequel of this proof. Remember that it is a best response for the Principal to retain the Agent if, and only if, $Pr(\theta = \theta_H | (o_1, o_2); (\mathring{a}_1, \mathring{a}_2)) \geq \pi$.

Suppose first that the Agent exerts effort levels $(\mathring{a}_1, \mathring{a}_2)$, with $\mathring{a}_1, \mathring{a}_2 > 0$. We then have

$$Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (\mathring{a}_1, \mathring{a}_2)) = \frac{\theta_H^2 c_1 \mathring{a}_1 c_2 \mathring{a}_2 \pi}{\theta_H^2 c_1 \mathring{a}_1 c_2 \mathring{a}_2 \pi + \theta_L^2 c_1 \mathring{a}_1 c_2 \mathring{a}_2 (1 - \pi)},$$

$$Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (\mathring{a}_1, \mathring{a}_2)) = \frac{(1-\theta_H c_1 \mathring{a}_1)(1-\theta_H c_2 \mathring{a}_2)\pi}{(1-\theta_H c_1 \mathring{a}_1)(1-\theta_H c_2 \mathring{a}_2)\pi + (1-\theta_L c_1 \mathring{a}_1)(1-\theta_L c_2 \mathring{a}_2)(1-\pi)},$$

and

$$Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_1, \mathring{a}_2)) = \frac{\theta_H c_i \mathring{a}_i (1-\theta_H c_j \mathring{a}_j)\pi}{\theta_H c_i \mathring{a}_i (1-\theta_H c_j \mathring{a}_j)\pi + \theta_L c_i \mathring{a}_i (1-\theta_L c_j \mathring{a}_j)(1-\pi)}.$$

Because $\theta_H > \theta_L > 0$ we have $Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_1 = s, o_2 = s); (a_1, a_2)) > \pi$ and $Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_1 = f, o_2 = f); (a_1, a_2)) < \pi$. Consequently, the Principal's best response is to retain upon observing $(o_1 = s, o_2 = s)$, and to dismiss upon observing $(o_1 = f, o_2 = f)$. In turn, $Pr(\theta = \theta_H | (o_i = s, o_j = f); (\mathring{a}_1, \mathring{a}_2)) = Pr(\theta = \theta_H | (o_i = s, o_j = f); (a_1, a_2)) \geq \pi$ if, and only if, $\mathring{a}_j < \frac{1}{c_j(\theta_H + \theta_L)}$ (respectively $a_j < \frac{1}{c_j(\theta_H + \theta_L)}$). Consequently, the Principal's best response is to retain upon observing $(o_i = s, o_j = f)$ for all $i = 1, 2$, $j \neq i$, if, and only if, $\mathring{a}_j < \frac{1}{c_j(\theta_H + \theta_L)}$ (respectively $a_j < \frac{1}{c_j(\theta_H + \theta_L)}$) for all $j = 1, 2$.

Define $T := (\pi\theta_H + (1-\pi)\theta_L)$ and suppose the Principal uses the moderate retention rule. The Agent then chooses a_1, a_2, so as to maximize $(c_1 T a_1 + c_2 T a_2 - T^2 c_1 a_1 c_2 a_2)B - a_1^2 - a_2^2$ subject to the constraints that $a_1, a_2 \geq 0$.

Using the Kuhn-Tucker approach, we set up the Lagrangean

$$L(a_1, a_2, \lambda_1, \lambda_2) = (c_1 T a_1 + c_2 T a_2 - T^2 c_1 a_1 c_2 a_2)B - a_1^2 - a_2^2 + \lambda_1 a_1 + \lambda_2 a_2.$$

Note that if, $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{B}T}$, the objective function, and the inequality constraints are concave in (a_1, a_2). Thus, the first order conditions are necessary and sufficient for a maximum. The first order conditions are:

$$\frac{\partial L}{\partial a_1} = c_1 T(1 - T c_2 a_2)B - 2a_1 + \lambda_1 = 0 \tag{1}$$

$$\frac{\partial L}{\partial a_2} = c_2 T(1 - Tc_1 a_1)B - 2a_2 + \lambda_2 = 0 \tag{2}$$

$$\lambda_1 a_1 = 0, \quad \lambda_2 a_2 = 0, \quad a_1, a_2 \geq 0, \quad \lambda_1, \lambda_2 \geq 0.$$

(a) Case 1: WLOG suppose $\lambda_1 > 0$, which implies, by $\lambda_1 a_1 = 0$, that $a_1 = 0$. Solving equation (1) for λ_1, given that $a_1 = 0$, we find $\lambda_1 = -c_1 T(1 - Tc_2 a_2)B$ which is greater or equal to 0 if, and only if, $a_2 \geq \frac{1}{Tc_2}$.

In turn, solving equation (2) for a_2, given $a_1 = 0$, we get $a_2 = \frac{BTc_2 + \lambda_2}{2}$. Because, $\lambda_2 \geq 0$ and $BTc_2 > 0$, we have $a_2 > 0$. $\lambda_2 a_2 = 0$, in turn, implies $\lambda_2 = 0$ and thus $a_2 = \frac{BTc_2}{2}$. If $c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, we have $a_2 = \frac{Bc_2 T}{2} < \frac{1}{c_2 T}$. Consequently, for $c_2 < \tilde{c}$, there is no solution in which $\lambda_1 > 0$. A similar argument shows that there is no solution with $\lambda_2 > 0$.

(b) Case 2: Hence, we now consider the case $\lambda_1 = \lambda_2 = 0$. Solving equations (1) and (2) for a_1 and a_2, we get

$$a_1 = \frac{Bc_1 T(2 - Bc_2^2 T^2)}{4 - B^2 c_1^2 c_2^2 T^4}, \quad a_2 = \frac{Bc_2 T(2 - Bc_1^2 T^2)}{4 - B^2 c_1^2 c_2^2 T^4}.$$

Notice that if $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, we have $2 - Bc_2^2 T^2 < 0$, $2 - Bc_1^2 T^2 < 0$, and $4 - B^2 c_1^2 c_2^2 T^4 < 0$. Consequently, for $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, we have $a_1, a_2 > 0$. We conclude that, if $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$,

$$a_1 = \frac{Bc_1 T(2 - Bc_2^2 T^2)}{4 - B^2 c_1^2 c_2^2 T^4}, \quad a_2 = \frac{Bc_2 T(2 - Bc_1^2 T^2)}{4 - B^2 c_1^2 c_2^2 T^4},$$

is the Agent's best response to the moderate retention rule.

Remember that the moderate retention rule, in turn, is a best response to the Agent's effort allocation if, and only if, $a_i < \frac{1}{c_i(\theta_H + \theta_L)}$ for all $i = 1, 2$. Note that, in any equilibrium in which the Agent exerts positive effort on both tasks, a_1

and a_2 are strictly increasing in c_1, and c_2, respectively. Hence, if c_1, c_2, are sufficiently low, we have $a_i < \frac{1}{c_i(\theta_H+\theta_L)}$ for all $i = 1, 2$. Consequently, there exists $\tilde{c} \leq \frac{\sqrt{2}}{\sqrt{BT}}$, such that if $c_1, c_2 \leq \tilde{c}$ the moderate incentives equilibrium is the unique equilibrium in which the Agent exerts effort on both tasks.

We now show that if $c_1, c_2 \leq \tilde{c}$, the moderate incentives equilibrium maximizes the Principal's welfare. To do so we now derive the effort level of the Agent in the equilibrium in which the Agent exerts effort solely on task i. Given such an effort allocation, the Principal retains the Agent if, and only if, $o_i = s$. The Agent thus maximizes $a_i c_i TB - a_i^2$ subject to the constraint that $a_i \geq 0$, which yields $a_i = \frac{BTc_i}{2}$ as a maximizer. Simple, but tedious algebra, shows that if $c_1, c_2 < \frac{\sqrt{2}}{\sqrt{BT}}$, then $\frac{Bc_1T(2-Bc_2^2T^2)}{4-B^2c_1^2c_2^2T^4} + \frac{Bc_2T(2-Bc_1^2T^2)}{4-B^2c_1^2c_2^2T^4} > \frac{BTc_i}{2}$ for all $i = 1, 2$. Consequently, for $c_1, c_2 \leq \tilde{c}$ total effort is higher in the moderate incentives equilibrium than in any i^{th} task equilibrium.

2. Under unbundling, the Principal retains Agent A_i if, and only if $o_i = s$. Agent A_i thus maximizes $a_i c_i T B_i - a_i^2$, which yields $a_i = \frac{B_i c_i T}{2}$. Total effort under unbundling is thus equal to $\frac{B_1 c_1 T}{2} + \frac{(B-B_1)c_2 T}{2}$. Maximizing this total effort with respect to B_1, yields $(a_1 = 0, a_2 = \frac{Bc_2T}{2})$ if $c_1 \leq c_2$ and $(a_1 = \frac{Bc_1T}{2}, a_2 = 0)$ if $c_1 > c_2$. Notice that maximal total effort under unbundling is thus equal to effort in the $i-th$ task equilibrium under bundling. It follows that, for $c_1, c_2 \leq \tilde{c}$ total effort is higher in the moderate incentives equilibrium under bundling than in the optimal (B_1, B_2) allocation equilibrium under unbundling.

□

9 Robustness: n tasks

In the model studied in the paper, we assume that there are two policy areas. In this subsection, we consider the robustness of the arguments in favor of bundling when there are n policy areas. As the purpose of this analysis is to consider the robustness of the arguments with respect to bundling, we suppress the full characterization. Moreover, to keep the analysis self-contained we restrict attention to the symmetric case where $p_i^H = p_H$ and $p_i^L = p_L$ and hence $p_i = \pi p_H + (1-\pi)p_L := p$ for all $i \in \{1, \ldots, n\}$.

Note that a generalization of Proposition 3 in the paper is obviously true in an n tasks setting as well.

As before, the Agent A_i exerts effort $a_i = 1$ under unbundling if, and only if, $pB_i - k \geq 0$. It follows that there exists a vector of benefits of holding office $(B_1, \ldots, B_n)$ for which effort on all n tasks can be sustained in equilibrium under unbundling if, and only if, $B \geq \frac{nk}{p}$.

We now derive conditions under which effort in all n areas can be sustained in equilibrium under bundling. Note that with n areas, there are n different retention rules that can sustain effort in all n areas, albeit for different parameter values p_H, p_L, π, k, B: the Principal retains the Agent if, and only if, there is success on at least j tasks, with $j \in \{1, \ldots, n\}$. To focus on robustness of our argument we only look at the two extremes: (1) the equilibrium, where the Principal retains the Agent if, and only if, there is success in all n areas, and (2) the equilibrium, where the Principal retains the Agent if, and only if, there is success in at least one area.

Proposition A. 6. *On the equilibrium path of play under bundling, the Agent chooses to exert effort in all n areas if the pair of policy-making environment and beliefs satisfies either of the following two sets of conditions:*

1. *(a) the complexity of each task is sufficiently low, $p^n B - nk \geq 0$;*

   (b) the Principal expects the Agent to exert effort in all policy areas; and

(c) *the Principal's estimation of the Agent's competence decreases unless the outcome is success on all tasks,* $p_H^{n-1}(1-p_H) \leq p_L^{n-1}(1-p_L)$*;*

or

2. (a) *the complexity of each task is moderate,* $p(1-p)^{n-1}B - k \geq 0$*; and*

   (b) *the Principal's estimation of the Agent's competence increases when the outcome is success on at least one task,* $p_H(1-p_H)^{n-1} \geq p_L(1-p_L)^{n-1}$*.*

When conditions (1) hold, the Principal retains if, and only if, there is success in all policy areas, when conditions (2) hold, the Principal retains if, and only if, there is success in at least one policy area.

Proof. Suppose the Principal uses the strict retention rule, reelecting if, and only if, there is success in all n areas. We denote this retention rule r_s. Then,

$$U_A(a_1 = \cdots = a_n = 1, r_s) = p^n B - nk$$

$$U_A(\mathbf{a}|\exists i \in \{1,\ldots,n\} \text{ s.t. } a_i = 1 \text{ and } \exists j \neq i \in \{1,\ldots,n\} \text{ s.t. } a_j = 0, r_s) < -k$$

$$U_A(a_1 = \cdots = a_n = 0, r_s) = 0.$$

Consequently, the Agent best-responds to the strict retention rule by exerting effort on all n tasks, if, and only if, $p^n B - nk \geq 0$.

Suppose the Principal uses the moderate retention rule, reelecting if, and only if, there is success on at least one task.

Then,

$$U_A(a_1 = \cdots = a_n = 1, r_m) = [1-(1-p)^n]\, B - nk$$

$$U_A(\mathbf{a}|\exists! i \in \{1,\ldots,n\} \text{ s.t. } a_i = 0, r_m) = \left[1-(1-p)^{n-1}\right] B - (n-1)k$$

$$U_A(\mathbf{a}|a_1 = \cdots = a_i = 1, a_{i+1} = \cdots = a_n = 0, r_m) = \left[1 - (1-p)^i\right] B - ik$$

We have $U_A(a_1 = \cdots = a_n = 1, r_m) \geq U_A(\mathbf{a}|\exists! i \in \{1, \ldots, n\}$ s.t. $a_i = 0)$ if, and only if, $p(1-p)^{n-1}B - k \geq 0$.

Similarly,

$$U_A(a_1 = \cdots = a_n = 1, r_m) \geq U_A(\mathbf{a}|a_1 = \cdots = a_i = 1, a_{i+1} = \cdots = a_n = 0, r_m)$$

if, and only if,

$$\left[(1-p)^i - (1-p)^n\right] B - (n+i)k \geq 0$$

if, and only if,

$$(1-p)^i \left[1 - (1-p)^{n-i}\right] B - (n+i)k \geq 0$$

if, and only if,

$$(1-p)^i \left[p(1-p)^{n-i-1} + p(1-p)^{n-i-2} + p(1-p)^{n-i-3} + \ldots + p(1-p)^0\right] B - (n+i)k \geq 0$$

if, and only if,

$$p(1-p)^{n-1}B - k + p(1-p)^{n-2}B - k + \ldots + p(1-p)^i B - k \geq 0.$$

Finally, notice that if $p(1-p)^{n-1}B - k \geq 0$ then $p(1-p)^j B - k \geq 0$ for $j < n-1$. It follows that if $U_A(a_1 = \cdots = a_n = 1, r_m) \geq U_A(\mathbf{a}|\exists! i \in \{1, \ldots, n\}$ s.t. $a_i = 0, r_m)$ then $U_A(a_1 = \cdots = a_n = 1, r_m) \geq U_A(\mathbf{a}|a_1 = \cdots = a_i = 1, a_{i+1} = \cdots = a_n = 0, r_m)$ for all $i \in \{1, \ldots, n-2\}$. It follows that the Agent best-responds to the moderate retention rule by choosing to exert effort on all n tasks if, and only if, $p(1-p)^{n-1}B - k \geq 0$.

Suppose the Agent exerts effort on all n tasks. It is a best-response for the Principal to use the strict retention rule if the Principal updates downwards on the competence of the Agent upon observing failure on one task, i.e.

$$Pr(\theta = \theta_H | \mathbf{o} \text{ s.t. } \exists ! j \in \{1, \dots, n\} \text{ s.t. } o_j = f | \mathring{a}_1 = \dots = \mathring{a}_n = 1) = \frac{p_H^{n-1}(1-p_H)\pi}{p_H^{n-1}(1-p_H)\pi + p_L^{n-1}(1-p_L)(1-\pi)} \leq \pi$$

if, and only if,

$$p_H^{n-1}(1-p_H) \leq p_L^{n-1}(1-p_L).$$

Similarly, it is a best-response for the Principal to use the moderate retention rule if the Principal updates upwards on the competence of the Agent upon observing success on one task, i.e.

$$Pr(\theta = \theta_H | \mathbf{o} \text{ s.t. } \exists ! j \in \{1, \dots, n\} \text{ s.t. } o_j = s | \mathring{a}_1 = \dots = \mathring{a}_n = 1) = \frac{p_H(1-p_H)^{n-1}\pi}{p_H(1-p_H)^{n-1}\pi + p_L^{n-1}(1-p_L)(1-\pi)} \geq \pi$$

if, and only if,

$$p_H(1-p_H)^{n-1} \geq p_L(1-p_L)^{n-1}.$$

□

Proposition A. 7. *1. There is a set of intermediate levels of policy area complexities $(\underline{e}, \overline{e})$ such that for environments with those complexity levels, bundling has a strict incentive advantage over unbundling.*

2. If effort on all n tasks is sustained via the strict retention rule under bundling, then bundling does not have a strict incentive advantage over unbundling.

Proof. To show part 2 it suffices to show that $p^n B - nk \geq 0$ implies $B > \frac{nk}{p}$. This follows from $\frac{nk}{p^n} > \frac{nk}{p}$ which is immediate because $p < 1$.

Similarly to show part 1. it suffices to show that for all $n \geq 2$, there exists a range of values of $p \in (0,1)$ such that $\frac{nk}{p} > \frac{k}{p(1-p)^{n-1}}$. This is true if, and only if, $1 < n(1-p)^{n-1}$. Note that $n(1-p)^{n-1}$ is decreasing in p and is equal to 0 when $p = 1$ and equal to n when $p = 0$. By continuity of $n(1-p)^{n-1}$ there exists $\overline{p}$ such that $n(1-\overline{p})^{n-1} = 1$. Hence, for all $p < \overline{p}$, we have $\frac{nk}{p} > \frac{k}{p(1-p)^{n-1}}$. Now fix (B, k) and let $\underline{p}$ be the value for which $\underline{p}(1-\underline{p})^{n-1}B - nk = 0$ For B sufficiently high, we have $\underline{p} < \overline{p}$. Hence, for all $p \in (\underline{p}, \overline{p})$ there exists values of $B > 0$ for which effort on all tasks can be sustained in equilibrium under bundling but not under unbundling. □