Increasing Leverage: Judicial Review as a Democracy-Enhancing Institution

Supplemental Appendix

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Abstract

In the supplemental online appendix, we characterize all the perfect Bayesian equilibria of the games with and without judicial review and prove that the equilibria we focus on in the main body of the text are unique if $\gamma \in (1-\pi, \pi)$ and the unique ones to satisfy $D1$ if $\gamma \in [\pi, 2-\pi)$

Baseline model

Proposition 1. *The following strategies and beliefs constitute the equilibrium of the baseline model when $\gamma \in (1-\pi, 2-\pi)$: The congruent Legislatures choose $(p_1 = \omega_1, p_2 = \omega_2)$, $N_{\cdot,-1}$ chooses $(p_1 = \omega_1, p_2 = 1)$, while $N_{\cdot,1}$ chooses $(p_1 = \omega_1, p_2 = -1)$ with probability $\frac{1}{\alpha} - 1$ and $(p_1 \neq \omega_1, p_2 = -1)$ with probability $2 - \frac{1}{\alpha}$.*

$$r^*(p_1 = \omega_1, p_2 = 1) = 1,\ r^*(p_1 = \omega_1, p_2 = -1) = \tfrac{1}{\gamma+\pi},\ r^*(p_1 \neq \omega_1, p_2 = \cdot) = 0.$$

$$\mu(p_1 = \omega_1, p_2 = 1) = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)} > \pi,$$

$$\mu(p_1 = \omega_1, p_2 = -1) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \frac{1-\alpha}{\alpha}\alpha(1-\pi)} = \pi,$$

$$\mu(p_1 \neq \omega_1, p_2 = -1) = 0.$$

We let off-the-equilibrium-path beliefs satisfy $\mu(p_1 \neq \omega_1, p_2 = 1) < \pi$.

If $\gamma \in (1-\pi, \pi)$ then this equilibrium is the unique one. If $\gamma \in [\pi, 2-\pi)$ then it is the unique equilibrium that survives criterion D1.

In the appendix we prove that this pair of strategies and beliefs is indeed an equilibrium. In the supplemental appendix we now prove that the equilibrium specified in Proposition 1 is unique if $\gamma \in (1-\pi, \pi)$ and the unique one to survive D1 if $\gamma \in [\pi, 2-\pi)$.

Lemma 1. *If $\gamma \in (1-\pi, \pi)$ then the equilibrium described in Proposition 1 is unique.*

Proof. Note that if a congruent Legislature chooses $(p_1 = \omega_1, p_2 = \omega_2)$ it gets at least $2+\pi$. The payoff from any other policy vector is at most $2+\gamma$. As $\gamma < \pi$, it is optimal for a congruent Legislature to choose $(p_1 = \omega_1, p_2 = \omega_2)$ independent of the strategies of the remaining actors. Further note that $N_{\cdot,1}$ receives a payoff of at most $\gamma + 1$ when choosing $(p_1 = \omega_1, p_2 = 1)$, while it gets a payoff of at least $3-\pi$ when choosing $(p_1 \neq \omega_1, p_2 = -1)$. As $\gamma < \pi < 2-\pi$, $N_{\cdot,1}$ never plays $(p_1 = \omega_1, p_2 = 1)$. It follows that in any equilibrium the Voter must have beliefs concentrated on $\{C_{\cdot,1}, N_{\cdot,-1}\}$ when observing $(p_1 = \omega_1, p_2 = 1)$. As $\alpha > \frac{1}{2}$, we have $\mu(p_1 = \omega_1, p_2 = 1) = \frac{\alpha\pi}{\alpha\pi + \eta_{\cdot,-1}(p_1=\omega_1,p_2=1)(1-\alpha)(1-\pi)} > \pi$ for all $\eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1) \in [0,1]$ and thus $r^*(p_1 = \omega_1, p_2 = 1) = 1$.

Moreover, as $C_{\cdot,\cdot}$ never plays $(p_1 \neq \omega_1, p_2 = \cdot)$, if $N_{\cdot,\cdot}$ chooses $(p_1 \neq \omega_1, p_2 = \cdot)$ with positive probability, then $\mu(p_1 \neq \omega_1, p_2 = \cdot) = 0$, which implies that $r^*(p_1 \neq \omega_1, p_2 = \cdot) = 0$ as well. But then it must be the case in any equilibrium that $N_{\cdot,-1}$ chooses $(p_1 = \omega_1, p_2 = 1)$ as this yields a payoff of $2+\gamma$ while any other policy vector, if played with positive probability, yields at most a payoff of $3-\pi < 2+\gamma$. Similarly, there is no equilibrium in which $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) > 0$ as then $N_{\cdot,1}$ receives a payoff of $2-\pi$, while choosing $(p_1 \neq \omega_1, p_2 = -1)$ yields a payoff of at least $3-\pi$. Given that we previously showed that $(p_1 = \omega_1, p_2 = 1)$ is strictly dominated by $(p_1 \neq \omega_1, p_2 = -1)$ for $N_{\cdot,1}$ we conclude that in any equilibrium $N_{\cdot,1}$ chooses between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$.

Note however that if $N_{\cdot,1}$ chooses $(p_1 \neq \omega_1, p_2 = -1)$ deterministically then $\mu(p_1 = \omega_1, p_2 = -1) = 1$, which implies that in equilibrium $r(p_1 = \omega_1, p_2 = -1) = 1$ as well. But then $N_{\cdot,1}$ deviates to $(p_1 = \omega_1, p_2 = -1)$ as $N_{\cdot,1}$ then receives a payoff of $2+\gamma > 3-\pi$. Similarly, if $N_{\cdot,1}$ chooses $(p_1 = \omega_1, p_2 = -1)$ deterministically then $\mu(p_1 = \omega_1, p_2 = -1) =$

$\frac{(1-\alpha)\pi}{(1-\alpha)\pi+\alpha(1-\pi)} < \pi$ and $r(p_1 = \omega_1, p_2 = -1) = 0$ in equilibrium. But then $N_{\cdot,1}$ gets at least $3 - \pi > 2 - \pi$ when deviating to $(p_1 \neq \omega_1, p_2 = -1)$. It follows that in any equilibrium $N_{\cdot,1}$ must be mixing between $(p_1 \neq \omega_1, p_2 = -1)$ and $(p_1 = \omega_1, p_2 = -1)$, which requires that it is indifferent between these two strategies. This is the case if the Voter reelects with probability $\frac{1}{\gamma+\pi}$ upon observing $(p_1 = \omega_1, p_2 = -1)$ which to be optimal for the Voter requires $\mu(p_1 = \omega_1, p_2 = -1) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi+\eta_{\cdot,1}(p_1=\omega_1,p_2=-1)\alpha(1-\pi)} = \pi$ and thus $\eta^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = \frac{1}{\alpha} - 1$. □

The following lemmas show that if $\gamma \in [\pi, 2-\pi)$ then the strategy profile described in Proposition 1 is the unique equilibrium that survives criterion D1.

Lemma 2. *If $\gamma \in [\pi, 2-\pi)$ then the equilibrium described in proposition 1 survives criterion D1.*

Proof. Note that for any retention probability there exists a specification of Voter beliefs about the types of the Legislature that makes this retention probability a best-response of the Voter when she observes p. Now let $D(t, T, p)$ be the set of retention probabilities that make type t strictly prefer p to his equilibrium strategy policy vector and let $D^0(t, T, p)$ be the set of retention probabilities that make type t exactly indifferent. Finally, let $D^1(t, T, p) \equiv D(t, T, p) \cup D^0(t, T, p)$. Criterion D1 requires that if for some type t there exists a type t' such that $D^1(t, T, p) \subset D(t', T, p)$, then the Voter should not believe that she is facing a Legislature of type t when she observes p.

As $\gamma < 2 - \pi$, $C_{\cdot,-1}$ never plays $(p_1 \neq \omega_1, p_2 = 1)$ no matter what reelection probability is used by the Voter. Hence, $D^1(C_{\cdot,-1}, T, (p_1 \neq \omega_1, p_2 = 1)) = \emptyset$. In equilibrium, $C_{\cdot,1}$ receives a payoff of $3 + \gamma$ which is the highest possible payoff it can achieve in this game. Moreover, the highest possible payoff it could achieve if deviating to $(p_1 \neq \omega_1, p_2 = 1)$ is $2 + \gamma < 3 + \gamma$. Therefore, $D^1(C_{\cdot,1}, T, (p_1 \neq \omega_1, p_2 = 1)) = \emptyset$. Finally, in equilibrium $N_{\cdot,-1}$ receives a payoff of $2 + \gamma$ while deviating to $(p_1 \neq \omega_1, p_2 = 1)$ yields $2 + r(p_1 \neq \omega_1, p_2 =$

$1)(\gamma+1)+(1-r(p_1\neq\omega_1,p_2=1))(1-\pi)$. Thus, $D(N_{\cdot,-1},T,(p_1\neq\omega_1,p_2=1))=\{r(p_1\neq\omega_1,p_2=-1)\in[0,1]:r(p_1\neq\omega_1,p_2=1)>1-\frac{1}{\gamma+\pi}\}\neq\emptyset$. It follows that for criterion D1 to be satisfied out-of-equilibrium beliefs must be specified as $\mu(p_1\neq\omega_1,p_2=1)=0$. □

As $\gamma<2-\pi$ the policy vector that gives the Legislature a policy payoff of 0 in the first period is strictly dominated by the policy vector that gives the Legislature a policy payoff of 2 in the first period for any type $t\in T$.

Lemma 3. *There is no equilibrium in which $C_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=1)$ and $(p_1\neq\omega_1,p_2=1)$.*

Proof. Assume otherwise. As $C_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=1)$ and $(p_1\neq\omega_1,p_2=1)$ it must be indifferent between these two policy vectors which implies that $r(p_1\neq\omega_1,p_2=1)=r(p_1=\omega_1,p_2=1)+\frac{1}{\gamma+1-\pi}$. But then $N_{\cdot,-1}$ and $N_{\cdot,1}$ never play $(p_1=\omega_1,p_2=1)$. Thus $\mu(p_1=\omega_1,p_2=1)=1$ and $r(p_1=\omega_1,p_2=1)=1$ which contradicts $r(p_1\neq\omega_1,p_2=1)=r(p_1=\omega_1,p_2=1)+\frac{1}{\gamma+1-\pi}$. □

Lemma 4. *There is no equilibrium in which $C_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=1)$ and $(p_1=\omega_1,p_2=-1)$.*

Proof. Assume otherwise. Then *(i)* $r(p_1=\omega_1,p_2=1)=r(p_1=\omega_1,p_2=-1)-\frac{1}{\gamma+1+\pi}$. $r(p_1=\omega_1,p_2=1)<1$ implies in equilibrium that $\mu(p_1=\omega_1,p_2=1)\leq\pi$. As $N_{\cdot,1}$ never plays $(p_1=\omega_1,p_2=1)$ this can only be satisfied if $\eta_{\cdot,-1}(p_1=\omega_1,p_2=1)>0$ which implies that *(ii)* $r(p_1=\omega_1,p_2=1)\geq r(p_1\neq\omega_1,p_2=1)+\frac{1}{\gamma+\pi}$. *(i)* and *(ii)* imply that $r(p_1=\omega_1,p_2=-1)\geq\frac{1}{\gamma+\pi}+\frac{1}{\gamma+1-\pi}$. But this is impossible as $\gamma<2-\pi$ implies that $\frac{1}{\gamma+\pi}+\frac{1}{\gamma+1-\pi}>1$. □

Lemma 5. *The following pair of strategies and beliefs describe all the equilibria in which $C_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=-1)$ and $(p_1\neq\omega_1,p_2=1)$:*

$C_{\cdot,1}$ mixes between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = 1)$ with $\kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) \leq 2 - \frac{1}{\alpha}$, $C_{\cdot,-1}$ chooses $(p_1 = \omega_1, p_2 = -1)$, $N_{\cdot,-1}$ plays $(p_1 \neq \omega_1, p_2 = 1)$ and $N_{\cdot,1}$ mixes between $(p_1 = \omega_1, p_2 = -1)$, and $(p_1 \neq \omega_1, p_2 = 1)$, with probabilities $\eta^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = \kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - 1 + \frac{1}{\alpha}$ and $\eta^*_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) = 2 - \kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - \frac{1}{\alpha}$ respectively.

$$r^*(p_1 = \omega_1, p_2 = -1) = r^*(p_1 \neq \omega_1, p_2 = 1) \geq \frac{1}{\gamma + 1 - \pi},$$

$$r^*(p_1 = \omega_1, p_2 = 1) = r^*(p_1 \neq \omega_1, p_2 = -1) = 0.$$

$$\mu(\mathbf{p}_2) = \frac{(\kappa_{\cdot,1}(\mathbf{p}_2)\alpha + 1 - \alpha)\pi}{(\kappa_{\cdot,1}(\mathbf{p}_2)\alpha + 1 - \alpha)\pi + \eta_{\cdot,1}(\mathbf{p}_2)\alpha(1 - \pi)} = \pi,$$

$$\mu(\mathbf{p}_3) = \frac{(1 - \kappa_{\cdot,1}(\mathbf{p}_2))\alpha\pi}{(1 - \kappa_{\cdot,1}(\mathbf{p}_2))\alpha\pi + (\eta_{\cdot,1}(\mathbf{p}_3)\alpha + 1 - \alpha)(1 - \pi)} = \pi.$$

For any p not played with positive probability out-of-equilibrium beliefs satisfy $\mu(p) \leq \pi$.

Proof. $C_{\cdot,1}$ mixing between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = 1)$ requires that (1) $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = 1) \geq r(p_1 = \omega_1, p_2 = 1) + \frac{1}{\gamma+1-\pi}$. Hence $C_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = -1)$ or $(p_1 \neq \omega_1, p_2 = -1)$. Assume first that $\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1) > 0$. This is only the case if $r(p_1 \neq \omega_1, p_2 = -1) > r(p_1 = \omega_1, p_2 = -1)$. But then $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = 0$. It follows that $\mu(p_1 = \omega_1, p_2 = -1) = 1$ and thus $r(p_1 = \omega_1, p_2 = -1) = 1$ as well which contradicts $r(p_1 \neq \omega_1, p_2 = -1) > r(p_1 = \omega_1, p_2 = -1)$. It follows that $\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) = 1$.

(1) also implies that $N_{\cdot,-1}$ plays $(p_1 \neq \omega_1, p_2 = 1)$ or $(p_1 \neq \omega_1, p_2 = -1)$. Assume first that $\eta_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1) > 0$. Then $\mu(p_1 \neq \omega_1, p_2 = -1) = 0$ and thus $r(p_1 \neq \omega_1, p_2 = -1) = 0$ as well and $N_{\cdot,-1}$ wants to deviate to $(p_1 \neq \omega_1, p_2 = 1)$. It follows that $\eta_{\cdot,-1}(p_1 \neq \omega_1, p_2 = 1) = 1$.

Now consider the decision-making of $N_{\cdot,1}$. Assume first that $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = 0$.

Then $\mu(p_1 = \omega_1, p_2 = -1) = 1$ and thus $r(p_1 = \omega_1, p_2 = -1) = 1$. This in turn implies that $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = -1) = 0$ as otherwise $\mu(p_1 \neq \omega_1, p_2 = -1) = 0$ and thus $r(p_1 \neq \omega_1, p_2 = -1) = 0$ and $N_{\cdot,1}$ wants to deviate to $(p_1 = \omega_1, p_2 = -1)$. Notice however that if $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) = 1$ then $\mu(p_1 \neq \omega_1, p_2 = 1) = \frac{\kappa_{\cdot,1}(p_1 \neq \omega_1, p_2=1)\alpha\pi}{\kappa_{\cdot,1}(p_1 \neq \omega_1, p_2=1)\alpha\pi + 1 - \pi} < \pi$ and thus $r(p_1 \neq \omega_1, p_2 = 1) = 0$. But then $N_{\cdot,1}$ wants to deviate to $(p_1 = \omega_1, p_2 = -1)$ as well.

Hence, let us assume that $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) > 0$. This requires that $r(p_1 = \omega_1, p_2 = -1) > 0$ and thus $\mu(p_1 = \omega_1, p_2 = -1) = \frac{(\kappa_{\cdot,1}(p_1=\omega_1,p_2=-1)\alpha + 1 - \alpha)\pi}{(\kappa_{\cdot,1}(p_1=\omega_1,p_2=-1)\alpha+1-\alpha)\pi + \eta_{\cdot,1}(p_1=\omega_1,p_2=-1)\alpha(1-\pi)} \geq \pi$ which is the case if, and only if $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) \leq \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - 1 + \frac{1}{\alpha}$. Note that the Right-hand side of this last expression is always non-negative. Remember that for $C_{\cdot,1}$ to be willing to mix, we also need $r(p_1 \neq \omega_1, p_2 = 1) = r(p_1 = \omega_1, p_2 = -1) > 0$ and thus $\mu(p_1 \neq \omega_1, p_2 = 1) = \frac{(1-\kappa_{\cdot,1}(p_1=\omega_1,p_2=-1))\alpha\pi}{(1-\kappa_{\cdot,1}(p_1=\omega_1,p_2=-1))\alpha\pi + (\eta_{\cdot,1}(p_1\neq\omega_1,p_2=1)\alpha+1-\alpha)(1-\pi)} \geq \pi$ which is the case if, and only if $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) \leq 1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) + 1 - \frac{1}{\alpha}$. Note that this last expression is superior or equal to 0 if, and only if $\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) \leq 2 - \frac{1}{\alpha}$. Finally, note that in equilibrium $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - 1 + \frac{1}{\alpha}$ and $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) = 1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) + 1 - \frac{1}{\alpha}$. Assume this is not the case, then $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) + \eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) < 1$. Hence, $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = -1) > 0$. For this to be optimal $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = 1) = \frac{1}{\gamma+\pi}$ must hold, which, however implies that $\mu(p_1 = \omega_1, p_2 = -1) = \mu(p_1 \neq \omega_1, p_2 = 1) = \pi$. This, however, is the case if, and only if $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - 1 + \frac{1}{\alpha}$ and $\eta_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) = 1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) + 1 - \frac{1}{\alpha}$.

Finally, note that in the arguments above we established that in equilibrium $r^*(p_1 = \omega_1, p_2 = 1)$ and $r^*(p_1 \neq \omega_1, p_2 = -1)$ must be lower than 1. As these policy vectors are not played in equilibrium, we sustain the equilibrium by letting out-of-equilibrium beliefs satisfy $\mu(p) \leq \pi$. □

Lemma 6. *The equilibria identified in lemma 5 do not survive Criterion D1.*

Proof. $D^1(N_{\cdot,1}, T, (p_1 = \omega_1, p_2 = 1)) = \emptyset \subset D^1(N_{\cdot,-1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0,1] : r \geq r^*(p_1 = \omega_1, p_2 = -1) + \frac{1}{\gamma+\pi}\} \subset D(C_{\cdot,1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0,1] : r > r^*(p_1 = \omega_1, p_2 = -1) - \frac{1}{1+\gamma-\pi}\}$. So, to satisfy D1, out-of-equilibrium beliefs must be specified as $\mu(p_1 = \omega_1, p_2 = 1) = 1$. But then the Voter would reelect upon observing $(p_1 = \omega_1, p_2 = 1)$ leading $C_{\cdot,1}$ to deviate to $(p_1 = \omega_1, p_2 = 1)$. □

Lemma 7. *The following pairs of strategies and beliefs describe all the equilibria in which $C_{\cdot,1}$ plays $(p_1 = \omega_1, p_2 = 1)$ deterministically:*

1. *$N_{\cdot,1}$ plays $(p_1 \neq \omega_1, p_2 = -1)$, all other types of Legislature play $(p_1 = \omega_1, p_2 = 1)$. The Voter reelects if, and only if she observes $(p_1 = \omega_1, p_2 = 1)$.*

$$\mu(p_1 = \omega_1, p_2 = 1) = \frac{\pi}{\pi + (1-\alpha)(1-\pi)} > \pi.$$

$$\mu(p_1 \neq \omega_1, p_2 = -1) = 0.$$

For any p not played with positive probability out-of-equilibrium beliefs satisfy $\mu(p) \leq \pi$.

2. *See pair of strategies and beliefs described in Proposition 1.*

Proof. Assume $C_{\cdot,1}$ plays $(p_1 = \omega_1, p_2 = 1)$. This requires that $r(p_1 = \omega_1, p_2 = 1) \geq \max\{r(p_1 = \omega_1, p_2 = -1), r(p_1 \neq \omega_1, p_2 = 1)\} - \frac{1}{\gamma+1-\pi}$. This implies that $\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1) = 0$. To demonstrate this, assume otherwise. Then $r(p_1 \neq \omega_1, p_2 = -1) \geq r(p_1 = \omega_1, p_2 = -1) + \frac{1}{\gamma+1-\pi} \geq \frac{1}{\gamma+1-\pi}$. It follows that $N_{\cdot,1}$ would play $(p_1 \neq \omega_1, p_2 = -1)$; note that $N_{\cdot,1}$ cannot play $(p_1 \neq \omega_1, p_2 = 1)$ with positive probability as then $\mu(p_1 \neq \omega_1, p_2 = 1) = 0$ and thus $r(p_1 \neq \omega_1, p_2 = 1) = 0$. But then $N_{\cdot,1}$ has incentive to deviate to $(p_1 \neq \omega_1, p_2 = -1)$.). However, then $\mu(p_1 \neq \omega_1, p_2 = -1) = \frac{\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1)(1-\alpha)\pi}{\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1)(1-\alpha)\pi + \alpha(1-\pi)} < \pi$ for all $\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1) \in [0,1]$ and thus $r(p_1 \neq \omega_1, p_2 = -1) = 0$. $\Rightarrow\Leftarrow$

$\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1) = 0$ in turn implies that $\eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1) = 1$. Proceed by contradiction and assume that $\eta_{\cdot,-1}(p_1 \neq \omega_1, p_2 = \cdot) > 0$. But, then $\mu(p_1 \neq \omega_1, p_2 = \cdot) = 0$ and thus $r(p_1 \neq \omega_1, p_2 = \cdot) = 0$. But, then $\mu(p_1 = \omega_1, p_2 = 1) = \frac{[\alpha+\kappa_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)]\pi}{[\alpha+\kappa_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)]\pi+[\eta_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)](1-\pi)} > \pi$ for all $\eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1) \in [0,1]$, which implies that $r(p_1 = \omega_1, p_2 = 1) > \frac{1}{\gamma+\pi}$. This gives $N_{\cdot,-1}$ incentive to deviate to $(p_1 = \omega_1, p_2 = 1)$.

Now assume that $C_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = 1)$, then $r(p_1 = \omega_1, p_2 = 1) \geq r(p_1 = \omega_1, p_2 = -1) + \frac{1}{\gamma+1-\pi} > \frac{1}{\gamma+\pi}$. As $N_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = 1)$ this implies that $\mu(p \neq (p_1 = \omega_1, p_2 = 1)) = 0$ for any policy vector $p \in P$ that $N_{\cdot,1}$ is willing to play. But then it is optimal for $N_{\cdot,1}$ to play $(p_1 \neq \omega_1, p_2 = -1)$.

Next we assume that $C_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = -1)$. Then, $N_{\cdot,1}$ plays $(p_1 = \omega_1, p_2 = -1)$ with positive probability. We prove this by contradiction. If we assume this is not the case, then $\mu(p_1 = \omega_1, p_2 = -1) = 1$ and thus $r(p_1 = \omega_1, p_2 = -1) = 1$. Moreover, $\mu(p_1 \neq \omega_1, p_2 = \cdot) = 0$ for any policy vector $(p_1 \neq \omega_1, p_2 = \cdot)$ that $N_{\cdot,1}$ plays with positive probability. Similarly, by assumption, $\mu(p_1 \neq \omega_1, p_2 = \cdot) = 0$ for any policy vector $(p_1 \neq \omega_1, p_2 = \cdot)$ that is not played with positive probability in equilibrium. Hence, $r(p_1 \neq \omega_1, p_2 = 1) = r(p_1 \neq \omega_1, p_2 = -1) = 0$, leading $N_{\cdot,1}$ to want to deviate to $(p_1 = \omega_1, p_2 = -1)$. Note also that in this setting $\mu(p_1 = \omega_1, p_2 = 1) = \frac{\alpha\pi}{\alpha\pi+(1-\alpha)(1-\pi)} > \pi$ and thus $r(p_1 = \omega_1, p_2 = 1) = 1$. This in turn implies that $r(p_1 = \omega_1, p_2 = -1) > 0$ because $r(p_1 = \omega_1, p_2 = -1) \geq r(p_1 = \omega_1, p_2 = 1) - \frac{1}{\gamma+1-\pi}$. But then $\mu(p_1 = \omega_1, p_2 = -1) = \frac{(1-\alpha)\pi}{(1-\alpha)\pi+\eta_{\cdot,1}(p_1=\omega_1,p_2=-1)\alpha(1-\pi)} \geq \pi$ in equilibrium, which implies that $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) \leq \frac{1-\alpha}{\alpha} < 1$. Moreover, given that $r(p_1 = \omega_1, p_2 = -1) > 0$ and $r(p_1 \neq \omega_1, p_2 = 1) = 0$ if $N_{\cdot,1}$ plays $(p_1 \neq \omega_1, p_2 = 1)$ with positive probability, then $N_{\cdot,1}$ is mixing between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$. This in turn implies that $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = -1) + \frac{1}{\gamma+\pi} = \frac{1}{\gamma+\pi}$ as $r(p_1 \neq \omega_1, p_2 = -1) = 0$. Hence, $r(p_1 = \omega_1, p_2 = -1) \in (0,1)$ and this requires $\mu(p_1 = \omega_1, p_2 = -1) = \pi$. Hence, $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = \frac{1}{\alpha} - 1$.

Finally, note that there is no equilibrium in which $C_{\cdot,-1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 = \omega_1, p_2 = -1)$. This would imply that $r(p_1 = \omega_1, p_2 = -1) = 1 - \frac{1}{\gamma+1-\pi}$. As seen above this implies that $N_{\cdot,1}$ plays $(p_1 \neq \omega_1, p_2 = -1)$. But then as $C_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = -1)$ with positive probability $\mu(p_1 = \omega_1, p_2 = -1) = 1$ and thus $r(p_1 = \omega_1, p_2 = 1) = 1$. $\Rightarrow\Leftarrow$ $\square$

Lemma 8. *The equilibrium identified in lemma 7.1 does not survive Criterion D1.*

Proof. $D^1(N_{\cdot,1}, T, (p_1 = \omega_1, p_2 = -1)) = \emptyset \subset D^1(N_{\cdot,-1}, T, (p_1 = \omega_1, p_2 = -1)) = \{r \in [0,1] : r \geq \frac{1}{\gamma+\pi}\} \subset D(C_{\cdot,-1}, T, (p_1 = \omega_1, p_2 = -1)) = \{r \in [0,1] : r > 1 - \frac{1}{1+\gamma-\pi}\}$. Hence, to satisfy D1, out-of-equilibrium beliefs must be specified as $\mu(p_1 = \omega_1, p_2 = -1) = 1$. But, then the Voter reelects upon observing $(p_1 = \omega_1, p_2 = -1)$ and thus $C_{\cdot,-1}$ wants to deviate to $(p_1 = \omega_1, p_2 = -1)$. $\square$

Lemma 9. *The following pair of strategies and beliefs constitute the only equilibrium in which $C_{\cdot,1}$ plays $(p_1 = \omega_1, p_2 = -1)$ deterministically:*

$N_{\cdot,-1}$ plays $(p_1 \neq \omega_1, p_2 = 1)$, all the other types of the Legislature play $(p_1 = \omega_1, p_2 = -1)$. The Voter reelects if and only if she observes $(p_1 = \omega_1, p_2 = -1)$.

$$\mu(p_1 = \omega_1, p_2 = -1) = \frac{\pi}{\pi + \alpha(1-\pi)} > \pi.$$

$$\mu(p_1 \neq \omega_1, p_2 = 1) = 0.$$

For any p not played with positive probability out-of-equilibrium beliefs satisfy $\mu(p) \leq \pi$.

Proof. Assume that $\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) = 1$. This is a best-response if, and only if $r(p_1 = \omega_1, p_2 = -1) > r(p_1 = \omega_1, p_2 = 1)$. Therefore, $C_{\cdot,-1}$ never plays $(p_1 = \omega_1, p_2 = 1)$. Assume that $\kappa_{\cdot,-1}(p_1 \neq \omega_1, p_2 = -1) > 0$. This is a best-response if, and only if $r(p_1 \neq$

$\omega_1, p_2 = -1) > r(p_1 = \omega_1, p_2 = -1)$. So, $N_{\cdot,1}$ never plays $(p_1 = \omega_1, p_2 = -1)$. But, then $\mu(p_1 = \omega_1, p_2 = -1) = 1$ and $r(p_1 = \omega_1, p_2 = -1) = 1$, implying that $C_{\cdot,-1}$ would want to deviate to $(p_1 = \omega_1, p_2 = -1)$. It follows that $\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) = 1$.

Given that $C_{\cdot,1}$ and $C_{\cdot,-1}$ play $(p_1 = \omega_1, p_2 = -1)$, $\mu(p_1 = \omega_1, p_2 = -1) = \frac{\pi}{\pi+\eta_{\cdot,1}(p_1=\omega_1,p_2=-1)\alpha(1-\pi)}$, which is greater than π for all $\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1) \in [0, 1]$. Hence, $r(p_1 = \omega_1, p_2 = -1) = 1$. As $\mu(p_1 \neq \omega_1, p_2 = \cdot) = 0$ if $(p_1 \neq \omega_1, p_2 = \cdot)$ is played with positive probability by N, $N_{\cdot,1}$ plays $(p_1 = \omega_1, p_2 = -1)$ as well. Similarly, $\mu(p_1 = \omega_1, p_2 = 1) = 0$ if $(p_1 = \omega_1, p_2 = 1)$ is played with positive probability by N. Hence, $N_{\cdot,-1}$'s best response is to choose $(p_1 \neq \omega_1, p_2 = 1)$. □

Lemma 10. *The equilibrium identified in lemma 9 does not survive Criterion D1.*

Proof. $D^1(N_{\cdot,-1}, T, (p_1 = \omega_1, p_2 = 1)) = \emptyset \subset D^1(N_{\cdot,1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0, 1] : r \geq \frac{1}{\gamma+\pi}\} \subset D(C_{\cdot,1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0, 1] : r > 1 - \frac{1}{1+\gamma-\pi}\}$. To satisfy D1, then, out-of-equilibrium beliefs must be specified as $\mu(p_1 = \omega_1, p_2 = 1) = 1$. But then the Voter reelects upon observing $(p_1 = \omega_1, p_2 = 1)$, and $C_{\cdot,1}$ wants to deviate to $(p_1 = \omega_1, p_2 = 1)$. □

The equilibria described in Proposition **??** and lemmas 3, 5, and 7 exhaust all possible equilibria of the baseline model when $\gamma \in [\pi, 2 - \pi)$.

Model with Judiciary

Proposition 2. *The following strategies and beliefs constitute the equilibrium of the model with the judiciary when $\gamma > 1 - \pi$:*

$C_{\cdot,1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$, $C_{\cdot,-1}$ mixes between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$, $N_{\cdot,-1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and

$(p_1 \neq \omega_1, p_2 = 1)$, *and* $N_{\cdot,1}$ *mixes between all* $p \in P$.

In equilibrium the mixing probabilities need to satisfy the following equations:

$$
\begin{aligned}
\eta^*_{\cdot,1}(p_1 = \omega_1, p_2 = 1) &= \kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = 1) - \eta^*_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)\frac{1-\alpha}{\alpha} \\
\eta^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) &= \kappa^*_{\cdot,-1}(p_1 = \omega_1, p_2 = -1)\frac{1-\alpha}{\alpha} \\
\eta^*_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1) &= 2 - \frac{1}{\alpha} + \frac{1-\alpha}{\alpha}\eta^*_{\cdot,-1}(p_1 = \omega_1, p_2 = 1) - \kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \\
\eta^*_{\cdot,1}(p_1 \neq \omega_1, p_2 = -1) &= (1 - \kappa^*_{\cdot,-1}(p_1 = \omega_1, p_2 = -1))\frac{1-\alpha}{\alpha}
\end{aligned} \tag{1}
$$

$r^*(p_1 = \omega_1, p_2 = 1) = r^*(p_1 \neq \omega_1, p_2 = 1) = r^*(p_1 = \omega_1, p_2 = -1) + \frac{1}{\gamma+\pi} = r^*(p_1 \neq \omega_1, p_2 = -1) + \frac{1}{\gamma+\pi}$.

If the specified policy vector is played with positive probability:

$$
\begin{aligned}
\mu(\mathbf{p}_1) &= \frac{\kappa^*_{\cdot,1}(\mathbf{p}_1)\alpha\pi}{\kappa^*_{\cdot,1}(\mathbf{p}_1)\alpha\pi + \left[\eta^*_{\cdot,1}(\mathbf{p}_1)\alpha + \eta^*_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)\right](1-\pi)} = \pi, \\
\mu(\mathbf{p}_2) &= \frac{\kappa^*_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi}{\kappa^*_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi + \eta^*_{\cdot,1}(\mathbf{p}_2)\alpha(1-\pi)} = \pi, \\
\mu(\mathbf{p}_3) &= \frac{(1-\kappa^*_{\cdot,1}(\mathbf{p}_1))\alpha\pi}{(1-\kappa^*_{\cdot,1}(\mathbf{p}_1))\alpha\pi + \left[\eta^*_{\cdot,1}(\mathbf{p}_3)\alpha + (1-\eta^*_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)\right](1-\pi)} = \pi, \\
\mu(\mathbf{p}_4) &= \frac{(1-\kappa^*_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi}{(1-\kappa^*_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi + \eta^*_{\cdot,1}(\mathbf{p}_4)\alpha(1-\pi)} = \pi.
\end{aligned} \tag{2}
$$

In case one of these policy vectors is not played with positive probability we let off-the-equilibrium-path beliefs satisfy: $\mu(p) \leq \pi$.

If $\gamma \in (1-\pi, \pi)$, then this is the unique type of equilibria. If $\gamma \geq \pi$ then it is the unique type of equilibria to survive criterion D1.

In the appendix we prove that these pairs of strategies and beliefs are indeed perfect Bayesian equilibria. Here, in the supplemental appendix we prove that they are the unique type of equilibria if $\gamma \in (1-\pi, \pi)$ and the unique ones to survive D1 if $\gamma \geq \pi$.

Lemma 11. *If $\gamma \in (1-\pi, \pi)$, then the type of equilibria described in Proposition 2 are the only ones.*

Proof. Note that the lowest possible payoff that $C_{\cdot,1}$ receives from playing $(p_1 = \omega_1, p_2 = 1)$ is $\pi_J + 1 + \pi$, while the highest possible payoff that $C_{\cdot,1}$ receives from playing $(p_1 = \omega_1, p_2 = -1)$ is $\pi_J + \gamma + 1$. As $\gamma < \pi$, $C_{\cdot,1}$ never plays $(p_1 = \omega_1, p_2 = -1)$. Similar arguments show that $C_{\cdot,1}$ never plays $(p_1 \neq \omega_1, p_2 = -1)$ and that $C_{\cdot,-1}$ never plays $(p_1 = \cdot, p_2 = 1)$. Hence, in any equilibrium $C_{\cdot,1}$ plays $(p_1 = \omega_1, p_2 = 1)$ or $(p_1 \neq \omega_1, p_2 = 1)$ while $C_{\cdot,-1}$ chooses between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$.

Note that it is never the case in equilibrium that $N_{\cdot,-1}$ plays $(p_1 = \cdot, p_2 = -1)$ with positive probability. We prove this by contradiction. Assume WLOG that $N_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = -1)$ with positive probability. For this behavior to be optimal, it must be the case that $r^*(p_1 = \omega_1, p_2 = -1) > \max\{r^*(p_1 = \omega_1, p_2 = 1), r^*(p_1 \neq \omega_1, p_2 = 1)\}$. But then $N_{\cdot,1}$ never plays $(p_1 = \cdot, p_2 = 1)$. It follows that

$$\mu(p_1 = \omega_1, p_2 = 1) = \frac{\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha\pi}{\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha\pi + \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)(1-\alpha)(1-\pi)},$$

while

$$\mu(p_1 \neq \omega_1, p_2 = 1) = \frac{\kappa_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1)\alpha\pi}{\kappa_{\cdot,1}(p_1 \neq \omega_1, p_2 = 1)\alpha\pi + \eta_{\cdot,-1}(p_1 \neq \omega_1, p_2 = 1)(1-\alpha)(1-\pi)}.$$

Furthermore, note that it must be the case that $\mu(p_1 = \omega_1, p_2 = 1) > \pi$ or $\mu(p_1 \neq \omega_1, p_2 = 1) > \pi$. If this were not true then the following ought to be: $\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \leq \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)\frac{1-\alpha}{\alpha}$, and $1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \leq (1 - \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1))\frac{1-\alpha}{\alpha}$. But this is impossible and thus $r^*(p_1 = \omega_1, p_2 = 1) = 1$ or $r^*(p_1 \neq \omega_1, p_2 = 1) = 1$. This however contradicts $r^*(p_1 = \omega_1, p_2 = -1) > \max\{r^*(p_1 = \omega_1, p_2 = 1), r^*(p_1 \neq \omega_1, p_2 = 1)\}$. Hence, in any equilibrium $N_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = 1)$ or $(p_1 \neq \omega_1, p_2 = 1)$.

We now consider the behavior of $N_{\cdot,1}$. Note first that it is never the case that $N_{\cdot,1}$ mixes exclusively between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$. If it did so then it must be the case that either $\mu(p_1 = \omega_1, p_2 = -1) = 1$ or $\mu(p_1 \neq \omega_1, p_2 = -1) = 1$, which would imply that $r^*(p_1 = \omega_1, p_2 = -1) = 1$ or $r^*(p_1 \neq \omega_1, p_2 = -1) = 1$. But then $N_{\cdot,1}$ would want to deviate to $(p_1 = \omega_1, p_2 = 1)$ or to $(p_1 \neq \omega_1, p_2 = 1)$, depending. Similarly, it cannot be the case that $N_{\cdot,1}$ mixes exclusively between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$. Assume otherwise. Then:

$$\mu(\mathbf{p}_1) = \frac{\kappa_{\cdot,1}(\mathbf{p}_1)\alpha\pi}{\kappa_{\cdot,1}(\mathbf{p}_1)\alpha\pi + \eta_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)(1-\pi)},$$

$$\mu(\mathbf{p}_2) = \frac{\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi}{\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi + \eta_{\cdot,1}(\mathbf{p}_2)\alpha(1-\pi)},$$

$$\mu(\mathbf{p}_3) = \frac{(1-\kappa_{\cdot,-1}(\mathbf{p}_1))\alpha\pi}{(1-\kappa_{\cdot,-1}(\mathbf{p}_1))\alpha\pi + (1-\eta_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)(1-\pi)},$$

and

$$\mu(\mathbf{p}_4) = \frac{(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi}{(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi + (1-\eta_{\cdot,1}(\mathbf{p}_2))\alpha(1-\pi)}.$$

Note that this implies that $\mu(p_1 = \omega_1, p_2 = -1) < \pi$ or $\mu(p_1 \neq \omega_1, p_2 = -1) < \pi$. If this were not true the following equations would need to be satisfied:

$$\kappa_{\cdot,-1}(p_1=\omega_1,p_2=-1)\geq \eta_{\cdot,1}(p_1=\omega_1,p_2=-1)\frac{\alpha}{1-\alpha},$$
and
$$1-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=-1)\geq (1-\eta_{\cdot,1}(p_1=\omega_1,p_2=-1))\frac{\alpha}{1-\alpha}.$$

But this is impossible. It follows that $r(p_1=\omega_1,p_2=-1)=0$ or $r(p_1\neq\omega_1,p_2=-1)=0$. Similarly, as we showed above it must then be the case that $\mu(p_1=\omega_1,p_2=1)>\pi$ or $\mu(p_1\neq\omega_1,p_2=1)>\pi$ and thus $r(p_1=\omega_1,p_2=1)=1$ or $r(p_1\neq\omega_1,p_2=1)=1$. But then $N_{\cdot,1}$ wants to deviate either to $(p_1=\omega_1,p_2=1)$ or $(p_1\neq\omega_1,p_2=1)$.

Hence, let us assume that $N_{\cdot,1}$ is mixing between $(p_1=\cdot,p_2=1)$ and $(p_1=\cdot,p_2=-1)$. For this to be a best-response, it must be the case that $r^*(p_1=\omega_1,p_2=1)=r^*(p_1\neq\omega_1,p_2=1)=r^*(p_1=\omega_1,p_2=-1)+\frac{1}{\gamma+\pi}=r^*(p_1\neq\omega_1,p_2=-1)+\frac{1}{\gamma+\pi}$. From this we infer that $r^*(p_1=\omega_1,p_2=1)=r^*(p_1\neq\omega_1,p_2=1)>0$ and that $r^*(p_1=\omega_1,p_2=-1)=r^*(p_1\neq\omega_1,p_2=-1)<1$. Such reelection probabilities are a best-response for the Voter if, and only if the following inequalities are true:

$$\mu(\mathbf{p}_1)=\frac{\kappa_{\cdot,1}(\mathbf{p}_1)\alpha\pi}{\kappa_{\cdot,1}(\mathbf{p}_1)\alpha\pi+[\eta_{\cdot,1}(\mathbf{p}_1)\alpha+\eta_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)](1-\pi)}\geq\pi,$$

$$\mu(\mathbf{p}_2)=\frac{\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi}{\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi+\eta_{\cdot,1}(\mathbf{p}_2)\alpha(1-\pi)}\leq\pi,$$

$$\mu(\mathbf{p}_3)=\frac{(1-\kappa_{\cdot,1}(\mathbf{p}_1))\alpha\pi}{(1-\kappa_{\cdot,1}(\mathbf{p}_1))\alpha\pi+[\eta_{\cdot,1}(\mathbf{p}_3)\alpha+(1-\eta_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)](1-\pi)}\geq\pi,$$

$$\mu(\mathbf{p}_4)=\frac{(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi}{(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi+\eta_{\cdot,1}(\mathbf{p}_4)\alpha(1-\pi)}\leq\pi.$$

In turn, these inequalities are true if, and only if

$$\begin{aligned}
\eta_{\cdot,1}(p_1=\omega_1,p_2=1) &\leq \kappa_{\cdot,1}(p_1=\omega_1,p_2=1)-\eta_{\cdot,-1}(p_1=\omega_1,p_2=1)\frac{1-\alpha}{\alpha} \\
\eta_{\cdot,1}(p_1=\omega_1,p_2=-1) &\leq \kappa_{\cdot,-1}(p_1=\omega_1,p_2=-1)\frac{1-\alpha}{\alpha} \\
\eta_{\cdot,1}(p_1\neq\omega_1,p_2=1) &\leq 2-\frac{1}{\alpha}+\frac{1-\alpha}{\alpha}\eta_{\cdot,-1}(p_1=\omega_1,p_2=1)-\kappa_{\cdot,1}(p_1=\omega_1,p_2=1) \\
\eta_{\cdot,1}(p_1\neq\omega_1,p_2=-1) &\leq (1-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=-1))\frac{1-\alpha}{\alpha}.
\end{aligned} \tag{3}$$

Note finally that the right hand sides of inequalities 3 sum up to 1 which, as $\sum_{p\in P}\eta_{\cdot,1}(p)=1$ as well, implies that each of these inequalities holds with equality. See equations (1). □

We now prove that if $\gamma\in[\pi,2-\pi)$, then the type of equilibria identified in Proposition 2 is the unique one to survive Criterion D1.

Lemma 12. *There is no equilibrium in which $C_{\cdot,1}$ mixes between $(p_1=\cdot,p_2=1)$ and $(p_1=\cdot,p_2=-1)$.*

Proof. WLOG assume that $C_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=1)$ and $(p_1=\omega_1,p_2=-1)$. This is a best-response if, and only if $r(p_1=\omega_1,p_2=1)=r(p_1=\omega_1,p_2=-1)-\frac{1}{\gamma+1-\pi}<r(p_1=\omega_1,p_2=-1)-\frac{1}{\gamma+\pi}$. But then $C_{\cdot,-1}$, $N_{\cdot,1}$ and $N_{\cdot,-1}$ strictly prefer $(p_1=\omega_1,p_2=-1)$ to $(p_1=\omega_1,p_2=1)$. Hence, $\mu(p_1=\omega_1,p_2=1)=1$, so $r(p_1=\omega_1,p_2=1)=1$, which contradicts $r(p_1=\omega_1,p_2=1)=r(p_1=\omega_1,p_2=-1)-\frac{1}{\gamma+1-\pi}$. □

Lemma 13. *There is no equilibrium in which $C_{\cdot,-1}$ mixes between $(p_1=\cdot,p_2=1)$ and $(p_1=\cdot,p_2=-1)$.*

Proof. WLOG assume that $C_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=1)$ and $(p_1=\omega_1,p_2=-1)$. This is a best-response if, and only if $r(p_1=\omega_1,p_2=1)=r(p_1=\omega_1,p_2=-1)+\frac{1}{\gamma+1-\pi}>r(p_1=\omega_1,p_2=-1)+\frac{1}{\gamma+\pi}$. But then $C_{\cdot,1}$, $N_{\cdot,1}$ and $N_{\cdot,-1}$ strictly prefer $(p_1=\omega_1,p_2=1)$

to $(p_1 = \omega_1, p_2 = -1)$. Hence, $\mu(p_1 = \omega_1, p_2 = -1) = 1$ and thus $r(p_1 = \omega_1, p_2 = -1) = 1$, which contradicts $r(p_1 = \omega_1, p_2 = 1) = r(p_1 = \omega_1, p_2 = -1) + \frac{1}{\gamma+1-\pi}$. □

Lemma 14. *There is no equilibrium in which $N_{\cdot,-1}$ mixes between $(p_1 = \cdot, p_2 = 1)$ and $(p_1 = \cdot, p_2 = -1)$.*

Proof. WLOG assume that $N_{\cdot,-1}$ plays $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 = \omega_1, p_2 = -1)$ with positive probability. This is a best-response if, and only if $r(p_1 = \omega_1, p_2 = 1) = r(p_1 = \omega_1, p_2 = -1) - \frac{1}{\gamma+\pi} > r(p_1 = \omega_1, p_2 = -1) - \frac{1}{\gamma+1-\pi}$, $r(p_1 = \omega_1, p_2 = 1) \geq r(p_1 \neq \omega_1, p_2 = 1)$, and $r(p_1 = \omega_1, p_2 = -1) \geq r(p_1 \neq \omega_1, p_2 = -1)$. But then $C_{\cdot,1}$ and $N_{\cdot,1}$ never play $(p_1 = \omega_1, p_2 = 1)$ nor $(p_1 \neq \omega_1, p_2 = 1)$, while $C_{\cdot,1}$ never plays $(p_1 = \omega_1, p_2 = -1)$ nor $(p_1 \neq \omega_1, p_2 = -1)$. Moreover, $r(p_1 = \omega_1, p_2 = 1) = r(p_1 = \omega_1, p_2 = -1) - \frac{1}{\gamma+\pi}$ and $r(p_1 = \omega_1, p_2 = 1) \geq r(p_1 \neq \omega_1, p_2 = 1)$ imply that $r(p_1 = \omega_1, p_2 = 1), r(p_1 \neq \omega_1, p_2 = 1) < 1$. Such reelection probabilities however are a best-response of the Voter if, and only if:

$$\mu(p_1 = \omega_1, p_2 = 1) = \frac{\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha\pi}{\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha\pi + \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)(1-\alpha)(1-\pi)} \leq \pi,$$

and

$$\mu(p_1 \neq \omega_1, p_2 = 1) = \frac{(1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1))\alpha\pi}{(1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1))\alpha\pi + \eta_{\cdot,-1}(p_1 \neq \omega_1, p_2 = 1)(1-\alpha)(1-\pi)} \leq \pi.$$

For these inequalities to be satisfied it must be the case that

$$\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \leq \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)\frac{1-\alpha}{\alpha},$$

and

$$1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \leq \eta_{\cdot,-1}(p_1 \neq \omega_1, p_2 = 1)\frac{1-\alpha}{\alpha},$$

which is impossible. It follows that $\mu(p_1 = \omega_1, p_2 = 1)$ or $\mu(p_1 \neq \omega_1, p_2 = 1)$ is strictly greater than π. But then $r(p_1 = \omega_1, p_2 = 1)$ or $r(p_1 \neq \omega_1, p_2 = 1)$ is equal to 1. $\Rightarrow\Leftarrow$ $\square$

Lemma 15. *The following pairs of strategies and beliefs describe all the equilibria in which $C_{\cdot,1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$:*

1. *Any type of the Legislature mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$. The mixing probabilities satisfy:*

   $\kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = 1) - \eta^*_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha = \eta^*_{\cdot,-1}(p_1 = \omega_1, p_2 = 1) - \kappa^*_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)(1-\alpha)$.

   $r^*(p_1 = \omega_1, p_2 = 1) = r^*(p_1 \neq \omega_1, p_2 = 1) \geq \max\{r^*(p_1 = \omega_1, p_2 = -1) - \frac{1}{\gamma+1-\pi}, r^*(p_1 \neq \omega_1, p_2 = -1) - \frac{1}{\gamma+1-\pi}\}$.

   If the specified policy vector is played with positive probability

   $\mu(\mathbf{p}_1) = \frac{\left[\kappa^*_{\cdot,1}(\mathbf{p}_1)\alpha + \kappa^*_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)\right]\pi}{\left[\kappa^*_{\cdot,1}(\mathbf{p}_1)\alpha + \kappa^*_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)\right]\pi + \left[\eta^*_{\cdot,1}(\mathbf{p}_1)\alpha + \eta^*_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)\right](1-\pi)} = \pi,$

   $\mu(\mathbf{p}_3) = \frac{\left[(1-\kappa^*_{\cdot,1}(\mathbf{p}_1))\alpha + (1-\kappa^*_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)\right]\pi}{\left[(1-\kappa^*_{\cdot,1}(\mathbf{p}_1))\alpha + (1-\kappa^*_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)\right]\pi + \left[(1-\eta^*_{\cdot,1}(\mathbf{p}_1))\alpha + (1-\eta^*_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)\right](1-\pi)} = \pi.$

   For any p not played with positive probability out-of-equilibrium beliefs satisfy $\mu(p) \leq \pi$.

2. *See pair of strategies and beliefs described in Proposition 2.*

Proof. Assume $C_{\cdot,1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$. This is a best-response for $C_{\cdot,1}$ if, and only if $r(p_1 = \omega_1, p_2 = 1) = r(p_1 \neq \omega_1, p_2 = 1) \geq \max\{r(p_1 = \omega_1, p_2 = -1) - \frac{1}{\gamma+1-\pi}, r(p_1 \neq \omega_1, p_2 = -1) - \frac{1}{\gamma+1-\pi}\}$. By lemma 14 $C_{\cdot,-1}$ either mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$ or mixes between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$.

Let us first assume that $C_{\cdot,-1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$. This is a best-response for $C_{\cdot,-1}$ if, and only if $r(p_1 = \omega_1, p_2 = 1) = r(p_1 \neq \omega_1, p_2 = 1) \geq$

$\max\{r(p_1=\omega_1,p_2=-1)+\frac{1}{\gamma+1-\pi},r(p_1\neq\omega_1,p_2=-1)+\frac{1}{\gamma+1-\pi}\}>0$. But then it is optimal for $N_{\cdot,1}$ and $N_{\cdot,-1}$ to mix between $(p_1=\omega_1,p_2=1)$ and $(p_1\neq\omega_1,p_2=1)$ as well. Moreover, for the specified reelection probabilities to be a best-response for the Voter the following inequalities need to be satisfied:

$$\mu(\mathbf{p}_1)=\frac{[\kappa_{\cdot,1}(\mathbf{p}_1)\alpha+\kappa_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)]\pi}{[\kappa_{\cdot,1}(\mathbf{p}_1)\alpha+\kappa_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)]\pi+[\eta_{\cdot,1}(\mathbf{p}_1)\alpha+\eta_{\cdot,-1}(\mathbf{p}_1)(1-\alpha)](1-\pi)}\geq\pi,$$

$$\mu(\mathbf{p}_3)=\frac{[(1-\kappa_{\cdot,1}(\mathbf{p}_1))\alpha+(1-\kappa_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)]\pi}{[(1-\kappa_{\cdot,1}(\mathbf{p}_1))\alpha+(1-\kappa_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)]\pi+[(1-\eta_{\cdot,1}(\mathbf{p}_1))\alpha+(1-\eta_{\cdot,-1}(\mathbf{p}_1))(1-\alpha)](1-\pi)}\geq\pi,$$

$\mu(p_1=\omega_1,p_2=-1)\leq\pi$, and $\mu(p_1\neq\omega_1,p_2=-1)\leq\pi$.

The last two inequalities are satisfied by the assumption on out-of-equilibrium beliefs. The first two inequalities in turn are satisfied if, and only if: $(\kappa_{\cdot,1}(p_1=\omega_1,p_2=1)-\eta_{\cdot,1}(p_1=\omega_1,p_2=1))\alpha=(\eta_{\cdot,-1}(p_1=\omega_1,p_2=1)-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=1))(1-\alpha)$.

Let us now assume that $C_{\cdot,-1}$ mixes between $(p_1=\omega_1,p_2=-1)$ and $(p_1\neq\omega_1,p_2=-1)$. This is a best-response for $C_{\cdot,-1}$ if, and only if $r(p_1=\omega_1,p_2=-1)=r(p_1\neq\omega_1,p_2=-1)\geq\max\{r(p_1=\omega_1,p_2=1)-\frac{1}{\gamma+1-\pi},r(p_1\neq\omega_1,p_2=1)-\frac{1}{\gamma+1-\pi}\}>0$. By lemma 15, $N_{\cdot,-1}$ either mixes between $(p_1=\omega_1,p_2=1)$ and $(p_1\neq\omega_1,p_2=1)$ or mixes between $(p_1=\omega_1,p_2=-1)$ and $(p_1\neq\omega_1,p_2=-1)$. Assume first that it mixes between $(p_1=\omega_1,p_2=-1)$ and $(p_1\neq\omega_1,p_2=-1)$. For this to be a best-response it must be the case that $r(p_1=\omega_1,p_2=-1)=r(p_1\neq\omega_1,p_2=-1)\geq\max\{r(p_1=\omega_1,p_2=1)+\frac{1}{\gamma+\pi},r(p_1\neq\omega_1,p_2=1)+\frac{1}{\gamma+\pi}\}$, which implies that $r(p_1=\cdot,p_2=-1)>r(p_1=\cdot,p_2=1)$. It follows then that $N_{\cdot,1}$ mixes between $(p_1=\omega_1,p_2=-1)$ and $(p_1\neq\omega_1,p_2=-1)$ as well. For the Voter to be willing to choose $r(p_1=\omega_1,p_2=-1)=r(p_1\neq\omega_1,p_2=-1)>0$ it must be the case that

$$\mu(\mathbf{p}_2)=\frac{\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi}{\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\pi+(\eta_{\cdot,1}(\mathbf{p}_2)\alpha+\eta_{\cdot,-1}(\mathbf{p}_2)(1-\alpha))(1-\pi)}\geq\pi$$

and

$$\mu(\mathbf{p}_4) = \frac{(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi}{(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\pi + \left[(1-\eta_{\cdot,1}(\mathbf{p}_2))\alpha + (1-\eta_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\right](1-\pi)} \geq \pi.$$

This however requires

$$\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) \geq \eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1)\frac{\alpha}{1-\alpha} + \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = -1),$$

and

$$(1-\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1)) \geq (1-\eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1))\frac{\alpha}{1-\alpha} + 1 - \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = -1),$$

which is impossible. It follows that $r(p_1 = \omega_1, p_2 = -1)$ or $r(p_1 \neq \omega_1, p_2 = -1)$ is equal to 0 contradicting $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = -1) \geq \max\{r(p_1 = \omega_1, p_2 = 1) + \frac{1}{\gamma+\pi}, r(p_1 \neq \omega_1, p_2 = 1) + \frac{1}{\gamma+\pi}\}$. We conclude that $N_{\cdot,-1}$ mixes between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$.

We now consider the behavior of $N_{\cdot,1}$. That $N_{\cdot,1}$ is not only mixing between $(p_1 = \omega_1, p_2 = 1)$ and $(p_1 \neq \omega_1, p_2 = 1)$ is obvious. If it were then $\mu(p_1 = \omega_1, p_2 = 1) = 1$ or $\mu(p_1 \neq \omega_1, p_2 = 1) = 1$ and thus $r(p_1 = \omega_1, p_2 = -1) = 1$ or $r(p_1 \neq \omega_1, p_2 = -1) = 1$. But then $N_{\cdot,1}$ would want to deviate to $(p_1 = \omega_1, p_2 = 1)$ or to $(p_1 \neq \omega_1, p_2 = 1)$ respectively. Similarly, it cannot be the case that $N_{\cdot,1}$ is only mixing between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$. Assume otherwise. Then:

$$\mu(p_1 = \omega_1, p_2 = 1) = \frac{\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha\pi}{\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1)\alpha\pi + \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)(1-\alpha)(1-\pi)},$$

$$\mu(p_1 = \omega_1, p_2 = -1) = \frac{\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1)(1-\alpha)\pi}{\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1)(1-\alpha)\pi + \eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1)\alpha(1-\pi)},$$

$$\mu(p_1 \neq \omega_1, p_2 = 1) = \frac{(1-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=1))\alpha\pi}{(1-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=1))\alpha\pi + (1-\eta_{\cdot,-1}(p_1=\omega_1,p_2=1))(1-\alpha)(1-\pi)},$$

and

$$\mu(p_1 \neq \omega_1, p_2 = -1) = \frac{(1-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=-1))(1-\alpha)\pi}{(1-\kappa_{\cdot,-1}(p_1=\omega_1,p_2=-1))(1-\alpha)\pi + (1-\eta_{\cdot,1}(p_1=\omega_1,p_2=-1))\alpha(1-\pi)}.$$

Note that it must be the case that $\mu(p_1 = \omega_1, p_2 = -1) < \pi$ or $\mu(p_1 \neq \omega_1, p_2 = -1) < \pi$. If this were not true the following equations would need to be satisfied: $\kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) \geq \eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1)\frac{\alpha}{1-\alpha}$, and $1 - \kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) \geq (1 - \eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1))\frac{\alpha}{1-\alpha}$. This is impossible. It follows that $r(p_1 = \omega_1, p_2 = -1) = 0$ or $r(p_1 \neq \omega_1, p_2 = -1) = 0$. Similarly, it must be the case that $\mu(p_1 = \omega_1, p_2 = 1) > \pi$ or $\mu(p_1 \neq \omega_1, p_2 = 1) > \pi$. If this were not true then the following must hold: $\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \leq \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1)\frac{1-\alpha}{\alpha}$, and $1 - \kappa_{\cdot,1}(p_1 = \omega_1, p_2 = 1) \leq (1 - \eta_{\cdot,-1}(p_1 = \omega_1, p_2 = 1))\frac{1-\alpha}{\alpha}$. Again, this is impossible, and thus $r(p_1 = \omega_1, p_2 = 1) = 1$ or $r(p_1 \neq \omega_1, p_2 = 1) = 1$. But, then $N_{\cdot,1}$ wants to deviate either to $(p_1 = \omega_1, p_2 = 1)$ or to $(p_1 \neq \omega_1, p_2 = -1)$.

Proceeding as in the proof of lemma 11 yields the result. □

Lemma 16. *The equilibria identified in lemma 15.1 do not survive Criterion D1.*

Proof. $D^1(N_{\cdot,-1}, T, (p_1 = \omega_1, p_2 = -1)) = \{r \in [0,1] : r \geq r^*(p_1 = \cdot, p_2 = 1) + \frac{1}{\gamma+\pi}\} \subset D^1(N_{\cdot,1}, T, (p_1 = \omega_1, p_2 = -1)) = \{r \in [0,1] : r \geq r^*(p_1 = \cdot, p_2 = 1) - \frac{1}{\gamma+\pi}\} \subset D(C_{\cdot,-1}, T, p_1 = \omega_1, p_2 = -1) = \{r \in [0,1] : r > r^*(p_1 = \cdot, p_2 = 1) - \frac{1}{\gamma+1-\pi}\}$. Hence, to satisfy D1 out-of-equilibrium beliefs must be specified as $\mu(p_1 = \cdot, p_2 = -1) = 1$. But then the Voter reelects upon observing $(p_1 = \cdot, p_2 = -1)$. Hence $U_{C_{\cdot,-1}}(p_1 = \cdot, p_2 = -1) = 2 + \pi_J + \gamma > \pi_J + r^*(p_1 = \cdot, p_2 = 1)(\gamma + 1) + (1 - r^*(p_1 = \cdot, p_2 = 1))\pi$ and $C_{\cdot,-1}$ deviates to $(p_1 = \cdot, p_2 = -1)$. □

Lemma 17. *The following pairs of strategies and beliefs describe all the equilibria in which*

$C_{\cdot,1}$ *mixes between* $(p_1 = \omega_1, p_2 = -1)$ *and* $(p_1 \neq \omega_1, p_2 = -1)$*:*

Any type of the Legislature mixes between $(p_1 = \omega_1, p_2 = -1)$ *and* $(p_1 \neq \omega_1, p_2 = -1)$*. The mixing probabilities satisfy* $\kappa^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - \eta^*_{\cdot,1}(p_1 = \omega_1, p_2 = -1)\alpha = \eta^*_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) - \kappa^*_{\cdot,-1}(p_1 = \omega_1, p_2 = -1)(1-\alpha)$.

$r^*(p_1 = \omega_1, p_2 = -1) = r^*(p_1 \neq \omega_1, p_2 = -1) \geq \max\{r^*(p_1 = \omega_1, p_2 = 1) + \frac{1}{\gamma+1-\pi}, r^*(p_1 \neq \omega_1, p_2 = 1) + \frac{1}{\gamma+1-\pi}\}$.

If the specified policy vector is played with positive probability:

$$\mu(\mathbf{p}_2) = \frac{\left[\kappa^*_{\cdot,1}(\mathbf{p}_2)\alpha+\kappa^*_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\right]\pi}{\left[\kappa^*_{\cdot,1}(\mathbf{p}_2)\alpha+\kappa^*_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\right]\pi+\left[\eta^*_{\cdot,1}(\mathbf{p}_2)\alpha+\eta^*_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)\right](1-\pi)} = \pi,$$

$$\mu(\mathbf{p}_4) = \frac{\left[(1-\kappa^*_{\cdot,1}(\mathbf{p}_2))\alpha+(1-\kappa^*_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\right]\pi}{\left[(1-\kappa^*_{\cdot,1}(\mathbf{p}_2))\alpha+(1-\kappa^*_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\right]\pi+\left[(1-\eta^*_{\cdot,1}(\mathbf{p}_2))\alpha+(1-\eta^*_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)\right](1-\pi)} = \pi.$$

For any p *not played with positive probability out-of-equilibrium beliefs satisfy* $\mu(p) \leq \pi$.

Proof. Assume $C_{\cdot,1}$ mixes between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$. This is a best-response by $C_{\cdot,1}$ if, and only if $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = -1) \geq \max\{r(p_1 = \omega_1, p_2 = 1) + \frac{1}{\gamma+1-\pi}, r(p_1 \neq \omega_1, p_2 = 1) + \frac{1}{\gamma+1-\pi}\} > 0$. This implies that $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = -1) > \max\{r(p_1 = \omega_1, p_2 = 1), r(p_1 \neq \omega_1, p_2 = 1)\}$. Hence, mixing between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$ is a best-response for $C_{\cdot,-1}$ and $N_{\cdot,1}$. Similarly, as $r(p_1 = \omega_1, p_2 = -1) = r(p_1 \neq \omega_1, p_2 = -1) > \max\{r(p_1 = \omega_1, p_2 = 1)+\frac{1}{\gamma+\pi}, r(p_1 \neq \omega_1, p_2 = 1)+\frac{1}{\gamma+\pi}\}$ mixing between $(p_1 = \omega_1, p_2 = -1)$ and $(p_1 \neq \omega_1, p_2 = -1)$ is a best-response for $N_{\cdot,-1}$ as well.

For the Voter to be willing to reelect with positive probability upon observing $(p_1 = \omega_1, p_2 = -1)$ or $(p_1 \neq \omega_1, p_2 = -1)$, it must be the case that:

$$\mu(\mathbf{p}_2) = \frac{[\kappa_{\cdot,1}(\mathbf{p}_2)\alpha+\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)]\pi}{[\kappa_{\cdot,1}(\mathbf{p}_2)\alpha+\kappa_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)]\pi+[\eta_{\cdot,1}(\mathbf{p}_2)\alpha+\eta_{\cdot,-1}(\mathbf{p}_2)(1-\alpha)](1-\pi)} \geq \pi,$$

and

$\mu(\mathbf{p}_4) = \frac{[(1-\kappa_{\cdot,1}(\mathbf{p}_2))\alpha+(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)]\pi}{[(1-\kappa_{\cdot,1}(\mathbf{p}_2))\alpha+(1-\kappa_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)]\pi+[(1-\eta_{\cdot,1}(\mathbf{p}_2))\alpha+(1-\eta_{\cdot,-1}(\mathbf{p}_2))(1-\alpha)](1-\pi)} \geq \pi.$

The first of these two inequalities holds if, and only if

$(\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - \eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1))\alpha \geq (\eta_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) - \kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1))(1-\alpha)$,

while the second holds if, and only if

$(\kappa_{\cdot,1}(p_1 = \omega_1, p_2 = -1) - \eta_{\cdot,1}(p_1 = \omega_1, p_2 = -1))\alpha \leq (\eta_{\cdot,-1}(p_1 = \omega_1, p_2 = -1) - \kappa_{\cdot,-1}(p_1 = \omega_1, p_2 = -1))(1-\alpha)$. □

Lemma 18. *The equilibria described in lemma 17 do not survive Criterion D1.*

Proof. $D^1(N_{\cdot,1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0,1] : r \geq r^*(p_1 = \cdot, p_2 = -1) + \frac{1}{\gamma+\pi}\} \subset D^1(N_{\cdot,-1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0,1] : r \geq r^*(p_1 = \cdot, p_2 = -1) - \frac{1}{\gamma+\pi}\} \subset D(C_{\cdot,1}, T, (p_1 = \omega_1, p_2 = 1)) = \{r \in [0,1] : r > r^*(p_1 = \cdot, p_2 = -1) - \frac{1}{1+\gamma-\pi}\}$. Hence, to satisfy D1 out-of-equilibrium beliefs must be specified as $\mu(p_1 = \cdot, p_2 = 1) = 1$. But then the Voter reelects upon observing $(p_1 = \cdot, p_2 = 1)$. Hence $U_{C_{\cdot,1}}(p_1 = \cdot, p_2 = 1) = 2 + \pi_J + \gamma > \pi_J + r^*(p_1 = \cdot, p_2 = -1)(\gamma+1) + (1 - r^*(p_1 = \cdot, p_2 = -1))\pi$ and $C_{\cdot,1}$ deviates to $(p_1 = \cdot, p_2 = 1)$. □

Note that the equilibria described in Proposition 2 and lemmas 15 and 17 exhaust all possible equilibria of the model with the judiciary when $\gamma \geq \pi$.